

Appendix A

Tables

Table A.1: Symmetry-adapted linear combinations of two-hole products

symmetry-adapted state	linear combination of simple products
1S	$\sqrt{\frac{1}{5}}[1, 10, 11, 12] - \sqrt{\frac{1}{5}}[3, 8, 11, 12] + \sqrt{\frac{1}{5}}[5, 6, 11, 12] + \sqrt{\frac{1}{5}}[4, 7, 11, 12] - \sqrt{\frac{1}{5}}[2, 9, 11, 12]$
$^1D_{-2}$	$\sqrt{\frac{2}{7}}[1, 6, 11, 12] - \sqrt{\frac{3}{7}}[3, 4, 11, 12] - \sqrt{\frac{2}{7}}[2, 5, 11, 12]$
$^1D_{-1}$	$\sqrt{\frac{3}{7}}[1, 8, 11, 12] - \sqrt{\frac{1}{14}}[3, 6, 11, 12] + \sqrt{\frac{1}{14}}[4, 5, 11, 12] - \sqrt{\frac{3}{7}}[2, 7, 11, 12]$
1D_0	$\sqrt{\frac{2}{7}}[1, 10, 11, 12] + \sqrt{\frac{1}{14}}[3, 8, 11, 12] - \sqrt{\frac{2}{7}}[5, 6, 11, 12] - \sqrt{\frac{1}{14}}[4, 7, 11, 12] - \sqrt{\frac{2}{7}}[2, 9, 11, 12]$
1D_1	$-\sqrt{\frac{3}{7}}[4, 9, 11, 12] + \sqrt{\frac{1}{14}}[6, 7, 11, 12] - \sqrt{\frac{1}{14}}[5, 8, 11, 12] + \sqrt{\frac{3}{7}}[3, 10, 11, 12]$
1D_2	$-\sqrt{\frac{2}{7}}[6, 9, 11, 12] - \sqrt{\frac{3}{7}}[7, 8, 11, 12] + \sqrt{\frac{2}{7}}[5, 10, 11, 12]$
$^1G_{-4}$	$[1, 2, 11, 12]$
$^1G_{-3}$	$-\sqrt{\frac{1}{2}}[2, 3, 11, 12] + \sqrt{\frac{1}{2}}[1, 4, 11, 12]$
$^1G_{-2}$	$\sqrt{\frac{3}{14}}[1, 6, 11, 12] + \sqrt{\frac{4}{7}}[3, 4, 11, 12] - \sqrt{\frac{3}{14}}[2, 5, 11, 12]$
$^1G_{-1}$	$\sqrt{\frac{1}{14}}[1, 8, 11, 12] + \sqrt{\frac{3}{7}}[3, 6, 11, 12] - \sqrt{\frac{3}{7}}[4, 5, 11, 12] - \sqrt{\frac{1}{14}}[2, 7, 11, 12]$
1G_0	$\sqrt{\frac{1}{70}}[1, 10, 11, 12] + \sqrt{\frac{8}{35}}[3, 8, 11, 12] + \sqrt{\frac{18}{35}}[5, 6, 11, 12] - \sqrt{\frac{8}{35}}[4, 7, 11, 12] - \sqrt{\frac{1}{70}}[2, 9, 11, 12]$
1G_1	$-\sqrt{\frac{1}{14}}[4, 9, 11, 12] - \sqrt{\frac{3}{7}}[6, 7, 11, 12] + \sqrt{\frac{3}{7}}[5, 8, 11, 12] + \sqrt{\frac{1}{14}}[3, 10, 11, 12]$
1G_2	$-\sqrt{\frac{3}{14}}[6, 9, 11, 12] + \sqrt{\frac{4}{7}}[7, 8, 11, 12] + \sqrt{\frac{3}{14}}[5, 10, 11, 12]$
1G_3	$\sqrt{\frac{1}{2}}[7, 10, 11, 12] - \sqrt{\frac{1}{2}}[8, 9, 11, 12]$
$^1G_{-4}$	$[9, 10, 11, 12]$

Table A.1: Symmetry-adapted linear combinations of two-hole products

symmetry-adapted state	linear combination of simple products
${}^3P_{-1}^{\uparrow\uparrow}$	$\sqrt{\frac{2}{5}}[1, 7, 11, 12] - \sqrt{\frac{3}{5}}[3, 5, 11, 12]$
${}^3P_0^{\uparrow\uparrow}$	$\sqrt{\frac{4}{5}}[1, 9, 11, 12] - \sqrt{\frac{1}{5}}[3, 7, 11, 12]$
${}^3P_1^{\uparrow\uparrow}$	$-\sqrt{\frac{2}{5}}[3, 9, 11, 12] + \sqrt{\frac{3}{5}}[5, 7, 11, 12]$
${}^3P_{-1}^{\uparrow\downarrow}$	$-\sqrt{\frac{1}{5}}[1, 8, 11, 12] + \sqrt{\frac{3}{10}}[3, 6, 11, 12] + \sqrt{\frac{3}{10}}[4, 5, 11, 12] - \sqrt{\frac{1}{5}}[2, 7, 11, 12]$
${}^3P_0^{\uparrow\downarrow}$	$-\sqrt{\frac{2}{5}}[1, 10, 11, 12] + \sqrt{\frac{1}{10}}[3, 8, 11, 12] + \sqrt{\frac{1}{10}}[4, 7, 11, 12] - \sqrt{\frac{2}{5}}[2, 9, 11, 12]$
${}^3P_1^{\uparrow\downarrow}$	$\sqrt{\frac{1}{5}}[4, 9, 11, 12] - \sqrt{\frac{3}{10}}[6, 7, 11, 12] - \sqrt{\frac{3}{10}}[5, 8, 11, 12] + \sqrt{\frac{1}{5}}[3, 10, 11, 12]$
${}^3P_{-1}^{\downarrow\downarrow}$	$\sqrt{\frac{2}{5}}[2, 8, 11, 12] - \sqrt{\frac{3}{5}}[4, 6, 11, 12]$
${}^3P_0^{\downarrow\downarrow}$	$\sqrt{\frac{4}{5}}[2, 10, 11, 12] - \sqrt{\frac{1}{5}}[4, 8, 11, 12]$
${}^3P_1^{\downarrow\downarrow}$	$-\sqrt{\frac{2}{5}}[4, 10, 11, 12] + \sqrt{\frac{3}{5}}[6, 8, 11, 12]$
${}^3F_{-3}^{\uparrow\uparrow}$	$[1, 3, 11, 12]$
${}^3F_{-2}^{\uparrow\uparrow}$	$[1, 5, 11, 12]$
${}^3F_{-1}^{\uparrow\uparrow}$	$\sqrt{\frac{3}{5}}[1, 7, 11, 12] + \sqrt{\frac{2}{5}}[3, 5, 11, 12]$
${}^3F_0^{\uparrow\uparrow}$	$\sqrt{\frac{4}{5}}[1, 9, 11, 12] + \sqrt{\frac{4}{5}}[3, 7, 11, 12]$
${}^3F_1^{\uparrow\uparrow}$	$-\sqrt{\frac{3}{5}}[3, 9, 11, 12] - \sqrt{\frac{2}{5}}[5, 7, 11, 12]$
${}^3F_2^{\uparrow\uparrow}$	$[5, 9, 11, 12]$
${}^3F_3^{\uparrow\uparrow}$	$[7, 9, 11, 12]$
${}^3F_{-3}^{\uparrow\downarrow}$	$\sqrt{\frac{1}{2}}[2, 3, 11, 12] + \sqrt{\frac{1}{2}}[1, 4, 11, 12]$
${}^3F_{-2}^{\uparrow\downarrow}$	$\sqrt{\frac{1}{2}}[2, 5, 11, 12] + \sqrt{\frac{1}{2}}[1, 6, 11, 12]$
${}^3F_{-1}^{\uparrow\downarrow}$	$\sqrt{\frac{3}{10}}[2, 7, 11, 12] + \sqrt{\frac{1}{5}}[4, 5, 11, 12] + \sqrt{\frac{1}{5}}[3, 6, 11, 12] + \sqrt{\frac{3}{10}}[1, 8, 11, 12]$
${}^3F_0^{\uparrow\downarrow}$	$-\sqrt{\frac{1}{10}}[1, 10, 11, 12] - \sqrt{\frac{2}{5}}[3, 8, 11, 12] - \sqrt{\frac{2}{5}}[4, 7, 11, 12] - \sqrt{\frac{1}{10}}[2, 9, 11, 12]$
${}^3F_1^{\uparrow\downarrow}$	$-\sqrt{\frac{3}{10}}[3, 10, 11, 12] - \sqrt{\frac{1}{5}}[5, 8, 11, 12] - \sqrt{\frac{1}{5}}[6, 7, 11, 12] - \sqrt{\frac{3}{10}}[4, 9, 11, 12]$
${}^3F_2^{\uparrow\downarrow}$	$-\sqrt{\frac{1}{2}}[5, 10, 11, 12] - \sqrt{\frac{1}{2}}[6, 9, 11, 12]$
${}^3F_3^{\uparrow\downarrow}$	$-\sqrt{\frac{1}{2}}[7, 10, 11, 12] - \sqrt{\frac{1}{2}}[8, 9, 11, 12]$
${}^3F_{-3}^{\downarrow\downarrow}$	$[2, 4, 11, 12]$
${}^3F_{-2}^{\downarrow\downarrow}$	$[2, 6, 11, 12]$
${}^3F_{-1}^{\downarrow\downarrow}$	$\sqrt{\frac{3}{5}}[2, 8, 11, 12] + \sqrt{\frac{2}{5}}[4, 6, 11, 12]$
${}^3F_0^{\downarrow\downarrow}$	$\sqrt{\frac{1}{5}}[2, 10, 11, 12] + \sqrt{\frac{4}{5}}[4, 8, 11, 12]$
${}^3F_1^{\downarrow\downarrow}$	$-\sqrt{\frac{3}{5}}[4, 10, 11, 12] - \sqrt{\frac{2}{5}}[6, 8, 11, 12]$
${}^3F_2^{\downarrow\downarrow}$	$-[6, 10, 11, 12]$
${}^3F_3^{\downarrow\downarrow}$	$-[8, 10, 11, 12]$

Table A.2: Symmetry-adapted linear combinations of three-hole products

symmetry-adapted state	linear combination of simple products
${}^2P_{-1}^\uparrow$	$\sqrt{\frac{2}{35}}[1, 4, 9, 11] + \sqrt{\frac{16}{105}}[1, 5, 8, 11] - \sqrt{\frac{8}{35}}[1, 3, 10, 11] + \sqrt{\frac{1}{105}}[1, 6, 7, 11] +$ $+\sqrt{\frac{2}{35}}[2, 3, 9, 11] - \sqrt{\frac{9}{70}}[3, 4, 7, 11] - \sqrt{\frac{9}{70}}[3, 5, 6, 11] - \sqrt{\frac{5}{21}}[2, 5, 7, 11]$
${}^2P_0^\uparrow$	$-\sqrt{\frac{8}{105}}[1, 5, 10, 11] + \sqrt{\frac{35}{105}}[1, 6, 9, 11] + \sqrt{\frac{9}{35}}[1, 7, 8, 11] - \sqrt{\frac{8}{105}}[2, 5, 9, 11] +$ $-\sqrt{\frac{9}{35}}[3, 4, 9, 11] - \sqrt{\frac{1}{210}}[3, 5, 8, 11] + \sqrt{\frac{2}{105}}[3, 6, 7, 11] - \sqrt{\frac{1}{210}}[4, 5, 7, 11]$
${}^2P_1^\uparrow$	$-\sqrt{\frac{2}{35}}[1, 8, 9, 11] - \sqrt{\frac{16}{105}}[4, 5, 9, 11] + \sqrt{\frac{8}{35}}[2, 7, 9, 11] - \sqrt{\frac{1}{105}}[3, 6, 9, 11] +$ $-\sqrt{\frac{2}{35}}[1, 7, 10, 11] - \sqrt{\frac{9}{70}}[3, 7, 8, 11] - \sqrt{\frac{9}{70}}[5, 6, 7, 11] + \sqrt{\frac{5}{21}}[3, 5, 10, 11]$
${}^2P_{-1}^\downarrow$	$\sqrt{\frac{2}{35}}[2, 3, 10, 11] + \sqrt{\frac{16}{105}}[2, 6, 7, 11] - \sqrt{\frac{8}{35}}[2, 4, 9, 11] + \sqrt{\frac{1}{105}}[2, 5, 8, 11] +$ $+\sqrt{\frac{2}{35}}[1, 4, 10, 11] + \sqrt{\frac{9}{70}}[3, 4, 8, 11] + \sqrt{\frac{9}{70}}[4, 5, 6, 11] - \sqrt{\frac{5}{21}}[1, 6, 8, 11]$
${}^2P_0^\downarrow$	$-\sqrt{\frac{8}{105}}[2, 6, 9, 11] + \sqrt{\frac{32}{105}}[2, 5, 10, 11] - \sqrt{\frac{9}{35}}[2, 7, 8, 11] - \sqrt{\frac{8}{105}}[1, 6, 10, 11] +$ $+\sqrt{\frac{9}{39}}[3, 4, 10, 11] - \sqrt{\frac{1}{210}}[4, 6, 7, 11] + \sqrt{\frac{2}{105}}[4, 5, 8, 11] - \sqrt{\frac{1}{210}}[3, 6, 8, 11]$
${}^2P_1^\downarrow$	$-\sqrt{\frac{2}{35}}[2, 7, 10, 11] - \sqrt{\frac{16}{105}}[3, 6, 10, 11] + \sqrt{\frac{8}{35}}[1, 8, 10, 11] - \sqrt{\frac{1}{105}}[4, 5, 10, 11] +$ $-\sqrt{\frac{2}{35}}[2, 8, 9, 11] + \sqrt{\frac{9}{70}}[4, 7, 8, 11] + \sqrt{\frac{9}{70}}[5, 6, 8, 11] + \sqrt{\frac{5}{21}}[4, 6, 9, 11]$
${}^2D_{-2}^\uparrow(1)$	$-\frac{1}{2}[1, 2, 9, 11] - \frac{1}{2}[1, 3, 8, 11] + \frac{1}{2}[1, 4, 7, 11] + \frac{1}{2}[1, 5, 6, 11]$
${}^2D_{-1}^\uparrow(1)$	$-\frac{1}{2}[1, 3, 10, 11] + \frac{1}{2}[2, 3, 9, 11] + \frac{1}{2}[3, 4, 7, 11] + \frac{1}{2}[3, 5, 6, 11]$
${}^2D_0^\uparrow(1)$	$-\frac{1}{2}[1, 5, 10, 11] + \frac{1}{2}[2, 5, 9, 11] + \frac{1}{2}[3, 5, 8, 11] - \frac{1}{2}[4, 5, 7, 11]$
${}^2D_1^\uparrow(1)$	$\frac{1}{2}[2, 7, 9, 11] - \frac{1}{2}[1, 7, 10, 11] + \frac{1}{2}[3, 7, 8, 11] + \frac{1}{2}[5, 6, 7, 11]$
${}^2D_2^\uparrow(1)$	$-\frac{1}{2}[1, 9, 10, 11] + \frac{1}{2}[4, 7, 9, 11] - \frac{1}{2}[3, 8, 9, 11] + \frac{1}{2}[5, 6, 9, 11]$
${}^2D_{-2}^\downarrow(1)$	$\frac{1}{2}[1, 2, 10, 11] - \frac{1}{2}[2, 4, 7, 11] + \frac{1}{2}[2, 3, 8, 11] - \frac{1}{2}[2, 5, 6, 11]$
${}^2D_{-1}^\downarrow(1)$	$-\frac{1}{2}[2, 4, 9, 11] + \frac{1}{2}[1, 4, 10, 11] - \frac{1}{2}[3, 4, 8, 11] - \frac{1}{2}[4, 5, 6, 11]$
${}^2D_0^\downarrow(1)$	$-\frac{1}{2}[2, 6, 9, 11] + \frac{1}{2}[1, 6, 10, 11] + \frac{1}{2}[4, 6, 7, 11] - \frac{1}{2}[3, 6, 8, 11]$
${}^2D_1^\downarrow(1)$	$\frac{1}{2}[1, 8, 10, 11] - \frac{1}{2}[2, 8, 9, 11] - \frac{1}{2}[4, 7, 8, 11] - \frac{1}{2}[5, 6, 8, 11]$
${}^2D_2^\downarrow(1)$	$\frac{1}{2}[2, 9, 10, 11] + \frac{1}{2}[3, 8, 10, 11] - \frac{1}{2}[4, 7, 10, 11] - \frac{1}{2}[5, 6, 10, 11]$
${}^2D_{-2}^\uparrow(2)$	$\sqrt{\frac{25}{84}}[1, 2, 9, 11] - \sqrt{\frac{3}{28}}[1, 3, 8, 11] - \sqrt{\frac{1}{84}}[1, 4, 7, 11] + \sqrt{\frac{3}{28}}[1, 5, 6, 11] +$ $+\sqrt{\frac{4}{21}}[2, 3, 7, 11] + \sqrt{\frac{2}{7}}[3, 4, 5, 11]$
${}^2D_{-1}^\uparrow(2)$	$-\sqrt{\frac{3}{28}}[1, 3, 10, 11] + \sqrt{\frac{4}{21}}[1, 4, 9, 11] - \sqrt{\frac{2}{7}}[1, 6, 7, 11] - \sqrt{\frac{1}{84}}[2, 3, 9, 11] +$ $+\sqrt{\frac{2}{7}}[2, 5, 7, 11] + \sqrt{\frac{1}{84}}[3, 4, 7, 11] - \sqrt{\frac{3}{28}}[3, 5, 6, 11]$
${}^2D_0^\uparrow(2)$	$-\sqrt{\frac{3}{28}}[1, 5, 10, 11] + \sqrt{\frac{2}{7}}[1, 7, 8, 11] + \sqrt{\frac{3}{28}}[2, 5, 9, 11] + \sqrt{\frac{2}{7}}[3, 4, 9, 11] +$ $-\sqrt{\frac{3}{28}}[3, 5, 8, 11] + \sqrt{\frac{3}{28}}[4, 5, 7, 11]$
${}^2D_1^\uparrow(2)$	$\sqrt{\frac{3}{28}}[2, 7, 9, 11] - \sqrt{\frac{4}{21}}[1, 8, 9, 11] + \sqrt{\frac{2}{7}}[3, 6, 9, 11] + \sqrt{\frac{1}{84}}[1, 7, 10, 11] +$ $-\sqrt{\frac{2}{7}}[3, 5, 10, 11] + \sqrt{\frac{1}{84}}[3, 7, 8, 11] - \sqrt{\frac{3}{28}}[5, 6, 7, 11]$
${}^2D_2^\uparrow(2)$	$\sqrt{\frac{25}{84}}[1, 9, 10, 11] + \sqrt{\frac{3}{28}}[4, 7, 9, 11] + \sqrt{\frac{1}{84}}[3, 8, 9, 11] + \sqrt{\frac{3}{28}}[5, 6, 9, 11] +$ $-\sqrt{\frac{4}{21}}[3, 7, 10, 11] + \sqrt{\frac{2}{7}}[5, 7, 8, 11]$

Table A.2: Symmetry-adapted linear combinations of three-hole products

symmetry-adapted state	linear combination of simple products
${}^2D_{-2}^\downarrow(2)$	$-\sqrt{\frac{25}{84}}[1, 2, 10, 11] - \sqrt{\frac{3}{28}}[2, 4, 7, 11] - \sqrt{\frac{1}{84}}[2, 3, 8, 11] - \sqrt{\frac{3}{28}}[2, 5, 6, 11] +$ $+\sqrt{\frac{4}{21}}[1, 4, 8, 11] - \sqrt{\frac{2}{7}}[3, 4, 6, 11]$
${}^2D_{-1}^\downarrow(2)$	$-\sqrt{\frac{3}{28}}[2, 4, 9, 11] + \sqrt{\frac{4}{21}}[2, 3, 10, 11] - \sqrt{\frac{2}{7}}[2, 5, 8, 11] - \sqrt{\frac{1}{84}}[1, 4, 10, 11] +$ $+\sqrt{\frac{2}{7}}[1, 6, 8, 11] - \sqrt{\frac{1}{84}}[3, 4, 8, 11] + \sqrt{\frac{3}{28}}[4, 5, 6, 11]$
${}^2D_0^\downarrow(2)$	$-\sqrt{\frac{3}{28}}[2, 6, 9, 11] - \sqrt{\frac{2}{7}}[2, 7, 8, 11] + \sqrt{\frac{3}{28}}[1, 6, 10, 11] - \sqrt{\frac{2}{7}}[3, 4, 10, 11] +$ $-\sqrt{\frac{3}{28}}[4, 6, 7, 11] + \sqrt{\frac{3}{28}}[3, 6, 8, 11]$
${}^2D_1^\downarrow(2)$	$\sqrt{\frac{3}{28}}[1, 8, 10, 11] - \sqrt{\frac{4}{21}}[2, 7, 10, 11] + \sqrt{\frac{2}{7}}[4, 5, 10, 11] + \sqrt{\frac{1}{84}}[2, 8, 9, 11] +$ $-\sqrt{\frac{2}{7}}[4, 6, 9, 11] - \sqrt{\frac{1}{84}}[4, 7, 8, 11] + \sqrt{\frac{3}{28}}[5, 6, 8, 11]$
${}^2D_2^\downarrow(2)$	$-\sqrt{\frac{25}{84}}[2, 9, 10, 11] + \sqrt{\frac{3}{28}}[3, 8, 10, 11] + \sqrt{\frac{1}{84}}[4, 7, 10, 11] - \sqrt{\frac{3}{28}}[5, 6, 10, 11] +$ $-\sqrt{\frac{4}{21}}[4, 8, 9, 11] - \sqrt{\frac{2}{7}}[6, 7, 8, 11]$
${}^2F_{-3}^\uparrow$	$\sqrt{\frac{1}{2}}[1, 2, 7, 11] - \sqrt{\frac{1}{12}}[1, 4, 5, 11] - \sqrt{\frac{1}{12}}[1, 3, 6, 11] + \sqrt{\frac{1}{3}}[2, 3, 5, 11]$
${}^2F_{-2}^\uparrow$	$\sqrt{\frac{1}{3}}[1, 2, 9, 11] - \sqrt{\frac{1}{12}}[1, 3, 8, 11] + \sqrt{\frac{1}{12}}[1, 4, 7, 11] - \sqrt{\frac{1}{2}}[3, 4, 5, 11]$
${}^2F_{-1}^\uparrow$	$-\sqrt{\frac{1}{30}}[1, 3, 10, 11] + \sqrt{\frac{3}{10}}[1, 4, 9, 11] - \sqrt{\frac{1}{20}}[1, 5, 8, 11] + \sqrt{\frac{1}{20}}[1, 6, 7, 11] +$ $-\sqrt{\frac{2}{15}}[2, 3, 9, 11] - \sqrt{\frac{2}{15}}[3, 4, 7, 11] + \sqrt{\frac{3}{10}}[3, 5, 6, 11]$
${}^2F_0^\uparrow$	$-\sqrt{\frac{1}{15}}[1, 5, 10, 11] + \sqrt{\frac{4}{15}}[1, 6, 9, 11] - \sqrt{\frac{1}{10}}[1, 7, 8, 11] - \sqrt{\frac{1}{15}}[2, 5, 9, 11] +$ $+\sqrt{\frac{1}{10}}[3, 4, 9, 11] + \sqrt{\frac{1}{15}}[3, 5, 8, 11] - \sqrt{\frac{4}{15}}[3, 6, 7, 11] + \sqrt{\frac{1}{15}}[4, 5, 7, 11]$
${}^2F_1^\uparrow$	$\sqrt{\frac{1}{30}}[2, 7, 9, 11] - \sqrt{\frac{3}{10}}[1, 8, 9, 11] + \sqrt{\frac{1}{20}}[4, 5, 9, 11] - \sqrt{\frac{1}{20}}[3, 6, 9, 11] +$ $+\sqrt{\frac{2}{15}}[1, 7, 10, 11] - \sqrt{\frac{2}{15}}[3, 7, 8, 11] + \sqrt{\frac{3}{10}}[5, 6, 7, 11]$
${}^2F_2^\uparrow$	$\sqrt{\frac{1}{3}}[1, 9, 10, 11] + \sqrt{\frac{1}{12}}[4, 7, 9, 11] - \sqrt{\frac{1}{12}}[3, 8, 9, 11] - \sqrt{\frac{1}{2}}[5, 7, 8, 11]$
${}^2F_3^\uparrow$	$\sqrt{\frac{1}{2}}[3, 9, 10, 11] + \sqrt{\frac{1}{12}}[5, 8, 9, 11] + \sqrt{\frac{1}{12}}[6, 7, 9, 11] - \sqrt{\frac{1}{3}}[5, 7, 10, 11]$
${}^2F_{-3}^\downarrow$	$-\sqrt{\frac{1}{2}}[1, 2, 8, 11] - \sqrt{\frac{1}{12}}[2, 3, 6, 11] - \sqrt{\frac{1}{12}}[2, 4, 5, 11] + \sqrt{\frac{1}{3}}[1, 4, 6, 11]$
${}^2F_{-2}^\downarrow$	$-\sqrt{\frac{1}{3}}[1, 2, 10, 11] - \sqrt{\frac{1}{12}}[2, 4, 7, 11] + \sqrt{\frac{1}{12}}[2, 3, 8, 11] + \sqrt{\frac{1}{2}}[3, 4, 6, 11]$
${}^2F_{-1}^\downarrow$	$-\sqrt{\frac{1}{30}}[2, 4, 9, 11] + \sqrt{\frac{3}{10}}[2, 3, 10, 11] - \sqrt{\frac{1}{20}}[2, 6, 7, 11] + \sqrt{\frac{1}{20}}[2, 5, 8, 11] +$ $-\sqrt{\frac{2}{15}}[1, 4, 10, 11] + \sqrt{\frac{2}{15}}[3, 4, 8, 11] - \sqrt{\frac{3}{10}}[4, 5, 6, 11]$
${}^2F_0^\downarrow$	$-\sqrt{\frac{1}{15}}[2, 6, 9, 11] + \sqrt{\frac{4}{15}}[2, 5, 10, 11] + \sqrt{\frac{1}{10}}[2, 7, 8, 11] - \sqrt{\frac{1}{15}}[1, 6, 10, 11] +$ $-\sqrt{\frac{1}{10}}[3, 4, 10, 11] + \sqrt{\frac{1}{15}}[4, 6, 7, 11] - \sqrt{\frac{4}{15}}[4, 5, 8, 11] + \sqrt{\frac{1}{15}}[3, 6, 8, 11]$
${}^2F_1^\downarrow$	$\sqrt{\frac{1}{30}}[1, 8, 10, 11] - \sqrt{\frac{3}{10}}[2, 7, 10, 11] + \sqrt{\frac{1}{20}}[3, 6, 10, 11] - \sqrt{\frac{1}{20}}[4, 5, 10, 11] +$ $+\sqrt{\frac{2}{15}}[2, 8, 9, 11] + \sqrt{\frac{2}{15}}[4, 7, 8, 11] - \sqrt{\frac{3}{10}}[5, 6, 8, 11]$
${}^2F_2^\downarrow$	$-\sqrt{\frac{1}{3}}[2, 9, 10, 11] + \sqrt{\frac{1}{12}}[3, 8, 10, 11] - \sqrt{\frac{1}{12}}[4, 7, 10, 11] + \sqrt{\frac{1}{2}}[6, 7, 8, 11]$
${}^2F_3^\downarrow$	$-\sqrt{\frac{1}{2}}[4, 9, 10, 11] + \sqrt{\frac{1}{12}}[6, 7, 10, 11] + \sqrt{\frac{1}{12}}[5, 8, 10, 11] - \sqrt{\frac{1}{3}}[6, 8, 9, 11]$
${}^2G_{-4}^\uparrow$	$-\sqrt{\frac{2}{5}}[1, 2, 5, 11] - \sqrt{\frac{3}{5}}[1, 3, 4, 11]$

Table A.2: Symmetry-adapted linear combinations of three-hole products

symmetry-adapted state	linear combination of simple products
${}^2G_{-3}^\uparrow$	$-\sqrt{\frac{3}{10}}[1, 2, 7, 11] + \sqrt{\frac{1}{20}}[1, 4, 5, 11] - \sqrt{\frac{9}{20}}[1, 3, 6, 11] + \sqrt{\frac{1}{5}}[2, 3, 5, 11]$
${}^2G_{-2}^\uparrow$	$-\sqrt{\frac{3}{35}}[1, 2, 9, 11] - \sqrt{\frac{27}{140}}[1, 3, 8, 11] - \sqrt{\frac{3}{140}}[1, 4, 7, 11] - \sqrt{\frac{12}{35}}[1, 5, 6, 11] +$ $+\sqrt{\frac{12}{35}}[2, 3, 7, 11] - \sqrt{\frac{1}{70}}[3, 4, 5, 11]$
${}^2G_{-1}^\uparrow$	$-\sqrt{\frac{3}{70}}[1, 3, 10, 11] - \sqrt{\frac{3}{70}}[1, 4, 9, 11] - \sqrt{\frac{7}{20}}[1, 5, 8, 11] + \sqrt{\frac{9}{140}}[1, 6, 7, 11] +$ $+\sqrt{\frac{6}{35}}[2, 3, 9, 11] + \sqrt{\frac{4}{35}}[2, 5, 7, 11] - \sqrt{\frac{6}{35}}[3, 4, 7, 11] - \sqrt{\frac{3}{70}}[3, 5, 6, 11]$
${}^2G_0^\uparrow$	$-\sqrt{\frac{1}{7}}[1, 5, 10, 11] - \sqrt{\frac{3}{14}}[1, 7, 8, 11] + \sqrt{\frac{1}{7}}[2, 5, 9, 11] - \sqrt{\frac{3}{14}}[3, 4, 9, 11] +$ $-\sqrt{\frac{1}{7}}[3, 5, 8, 11] + \sqrt{\frac{1}{7}}[4, 5, 7, 11]$
${}^2G_1^\uparrow$	$\sqrt{\frac{3}{70}}[2, 7, 9, 11] + \sqrt{\frac{3}{70}}[1, 8, 9, 11] + \sqrt{\frac{7}{20}}[4, 5, 9, 11] - \sqrt{\frac{9}{140}}[3, 6, 9, 11] +$ $-\sqrt{\frac{6}{35}}[1, 7, 10, 11] - \sqrt{\frac{4}{35}}[3, 5, 10, 11] - \sqrt{\frac{6}{35}}[3, 7, 8, 11] - \sqrt{\frac{3}{70}}[5, 6, 7, 11]$
${}^2G_2^\uparrow$	$-\sqrt{\frac{3}{35}}[1, 9, 10, 11] + \sqrt{\frac{27}{140}}[4, 7, 9, 11] + \sqrt{\frac{3}{140}}[3, 8, 9, 11] - \sqrt{\frac{12}{35}}[5, 6, 9, 11] +$ $-\sqrt{\frac{12}{35}}[3, 7, 10, 11] - \sqrt{\frac{1}{70}}[5, 7, 8, 11]$
${}^2G_3^\uparrow$	$-\sqrt{\frac{3}{10}}[3, 9, 10, 11] - \sqrt{\frac{1}{20}}[5, 8, 9, 11] + \sqrt{\frac{9}{20}}[6, 7, 9, 11] - \sqrt{\frac{1}{5}}[5, 7, 10, 11]$
${}^2G_4^\uparrow$	$-\sqrt{\frac{2}{5}}[5, 9, 10, 11] - \sqrt{\frac{3}{5}}[7, 8, 9, 11]$
${}^2G_{-4}^\downarrow$	$\sqrt{\frac{2}{5}}[1, 2, 6, 11] + \sqrt{\frac{3}{5}}[2, 3, 4, 11]$
${}^2G_{-3}^\downarrow$	$\sqrt{\frac{3}{10}}[1, 2, 8, 11] + \sqrt{\frac{1}{20}}[2, 3, 6, 11] - \sqrt{\frac{9}{20}}[2, 4, 5, 11] + \sqrt{\frac{1}{5}}[1, 4, 6, 11]$
${}^2G_{-2}^\downarrow$	$\sqrt{\frac{3}{35}}[1, 2, 10, 11] - \sqrt{\frac{27}{140}}[2, 4, 7, 11] - \sqrt{\frac{3}{140}}[2, 3, 8, 11] + \sqrt{\frac{12}{35}}[2, 5, 6, 11] +$ $+\sqrt{\frac{12}{35}}[1, 4, 8, 11] + \sqrt{\frac{1}{70}}[3, 4, 6, 11]$
${}^2G_{-1}^\downarrow$	$-\sqrt{\frac{3}{70}}[2, 4, 9, 11] - \sqrt{\frac{3}{70}}[2, 3, 10, 11] - \sqrt{\frac{7}{20}}[2, 6, 7, 11] + \sqrt{\frac{9}{140}}[2, 5, 8, 11] +$ $+\sqrt{\frac{6}{35}}[1, 4, 10, 11] + \sqrt{\frac{4}{35}}[1, 6, 8, 11] + \sqrt{\frac{6}{35}}[3, 4, 8, 11] + \sqrt{\frac{3}{70}}[4, 5, 6, 11]$
${}^2G_0^\downarrow$	$-\sqrt{\frac{1}{7}}[2, 6, 9, 11] + \sqrt{\frac{3}{14}}[2, 7, 8, 11] + \sqrt{\frac{1}{7}}[1, 6, 10, 11] + \sqrt{\frac{3}{14}}[3, 4, 10, 11] +$ $-\sqrt{\frac{1}{7}}[4, 6, 7, 11] + \sqrt{\frac{1}{7}}[3, 6, 8, 11]$
${}^2G_1^\downarrow$	$\sqrt{\frac{3}{70}}[1, 8, 10, 11] + \sqrt{\frac{3}{70}}[2, 7, 10, 11] + \sqrt{\frac{7}{20}}[3, 6, 10, 11] - \sqrt{\frac{9}{140}}[4, 5, 10, 11] +$ $-\sqrt{\frac{6}{35}}[2, 8, 9, 11] - \sqrt{\frac{4}{35}}[4, 6, 9, 11] + \sqrt{\frac{6}{35}}[4, 7, 8, 11] + \sqrt{\frac{3}{70}}[5, 6, 8, 11]$
${}^2G_2^\downarrow$	$\sqrt{\frac{3}{35}}[2, 9, 10, 11] + \sqrt{\frac{27}{140}}[3, 8, 10, 11] + \sqrt{\frac{3}{140}}[4, 7, 10, 11] + \sqrt{\frac{12}{35}}[5, 6, 10, 11] +$ $-\sqrt{\frac{12}{35}}[4, 8, 9, 11] + \sqrt{\frac{1}{70}}[6, 7, 8, 11]$
${}^2G_3^\downarrow$	$\sqrt{\frac{3}{10}}[4, 9, 10, 11] - \sqrt{\frac{1}{20}}[6, 7, 10, 11] + \sqrt{\frac{9}{20}}[5, 8, 10, 11] - \sqrt{\frac{1}{5}}[6, 8, 9, 11]$
${}^2G_4^\downarrow$	$\sqrt{\frac{2}{5}}[6, 9, 10, 11] + \sqrt{\frac{3}{5}}[7, 8, 10, 11]$
${}^2H_{-5}^\uparrow$	$[1, 2, 3, 11]$
${}^2H_{-4}^\uparrow$	$\sqrt{\frac{3}{5}}[1, 2, 5, 11] - \sqrt{\frac{2}{5}}[1, 3, 4, 11]$
${}^2H_{-3}^\uparrow$	$\sqrt{\frac{1}{5}}[1, 2, 7, 11] + \sqrt{\frac{8}{15}}[1, 4, 5, 11] - \sqrt{\frac{2}{15}}[1, 3, 6, 11] - \sqrt{\frac{2}{15}}[2, 3, 5, 11]$
${}^2H_{-2}^\uparrow$	$\sqrt{\frac{1}{30}}[1, 2, 9, 11] - \sqrt{\frac{1}{30}}[1, 3, 8, 11] + \sqrt{\frac{3}{10}}[1, 4, 7, 11] - \sqrt{\frac{3}{10}}[1, 5, 6, 11] +$

Table A.2: Symmetry-adapted linear combinations of three-hole products

symmetry-adapted state	linear combination of simple products
${}^2\text{H}_{-1}^\uparrow$	$-\sqrt{\frac{2}{15}}[2, 3, 7, 11] + \sqrt{\frac{1}{5}}[3, 4, 5, 11]$
${}^2\text{H}_0^\uparrow$	$-\sqrt{\frac{1}{210}}[1, 3, 10, 11] + \sqrt{\frac{8}{105}}[1, 4, 9, 11] - \sqrt{\frac{4}{35}}[1, 5, 8, 11] + \sqrt{\frac{9}{35}}[1, 6, 7, 11] +$ $-\sqrt{\frac{3}{70}}[2, 3, 9, 11] - \sqrt{\frac{1}{35}}[2, 5, 7, 11] + \sqrt{\frac{32}{105}}[3, 4, 7, 11] - \sqrt{\frac{6}{35}}[3, 5, 6, 11]$
${}^2\text{H}_1^\uparrow$	$-\sqrt{\frac{1}{42}}[1, 5, 10, 11] + \sqrt{\frac{2}{21}}[1, 6, 9, 11] - \sqrt{\frac{1}{7}}[1, 7, 8, 11] - \sqrt{\frac{1}{42}}[2, 5, 9, 11] +$ $+\sqrt{\frac{1}{7}}[3, 4, 9, 11] - \sqrt{\frac{2}{21}}[3, 5, 8, 11] + \sqrt{\frac{8}{21}}[3, 6, 7, 11] - \sqrt{\frac{2}{21}}[4, 5, 7, 11]$
${}^2\text{H}_2^\uparrow$	$\sqrt{\frac{1}{210}}[2, 7, 9, 11] - \sqrt{\frac{8}{105}}[1, 8, 9, 11] + \sqrt{\frac{4}{35}}[4, 5, 9, 11] - \sqrt{\frac{9}{35}}[3, 6, 9, 11] +$ $+\sqrt{\frac{3}{70}}[1, 7, 10, 11] + \sqrt{\frac{1}{35}}[3, 5, 10, 11] + \sqrt{\frac{32}{105}}[3, 7, 8, 11] - \sqrt{\frac{6}{35}}[5, 6, 7, 11]$
${}^2\text{H}_3^\uparrow$	$\sqrt{\frac{1}{30}}[1, 9, 10, 11] + \sqrt{\frac{1}{30}}[4, 7, 9, 11] - \sqrt{\frac{3}{10}}[3, 8, 9, 11] - \sqrt{\frac{3}{10}}[5, 6, 9, 11] +$ $+\sqrt{\frac{2}{15}}[3, 7, 10, 11] + \sqrt{\frac{1}{5}}[5, 7, 8, 11]$
${}^2\text{H}_4^\uparrow$	$\sqrt{\frac{1}{5}}[3, 9, 10, 11] - \sqrt{\frac{8}{15}}[5, 8, 9, 11] + \sqrt{\frac{2}{15}}[6, 7, 9, 11] + \sqrt{\frac{2}{15}}[5, 7, 10, 11]$
${}^2\text{H}_5^\uparrow$	$\sqrt{\frac{3}{5}}[5, 9, 10, 11] - \sqrt{\frac{2}{5}}[7, 8, 9, 11]$
${}^2\text{H}_{-5}^\downarrow$	$[7, 9, 10, 11]$
${}^2\text{H}_{-4}^\downarrow$	$[1, 2, 4, 11]$
${}^2\text{H}_{-3}^\downarrow$	$-\sqrt{\frac{3}{5}}[1, 2, 6, 11] + \sqrt{\frac{2}{5}}[2, 3, 4, 11]$
${}^2\text{H}_{-2}^\downarrow$	$-\sqrt{\frac{1}{5}}[1, 2, 8, 11] + \sqrt{\frac{8}{15}}[2, 3, 6, 11] - \sqrt{\frac{2}{15}}[2, 4, 5, 11] - \sqrt{\frac{2}{15}}[1, 4, 6, 11]$
${}^2\text{H}_{-1}^\downarrow$	$-\sqrt{\frac{1}{30}}[1, 2, 10, 11] - \sqrt{\frac{1}{30}}[2, 4, 7, 11] + \sqrt{\frac{3}{10}}[2, 3, 8, 11] + \sqrt{\frac{3}{10}}[2, 5, 6, 11] +$ $-\sqrt{\frac{2}{15}}[1, 4, 8, 11] - \sqrt{\frac{1}{5}}[3, 4, 6, 11]$
${}^2\text{H}_0^\downarrow$	$-\sqrt{\frac{1}{210}}[2, 4, 9, 11] + \sqrt{\frac{8}{105}}[2, 3, 10, 11] - \sqrt{\frac{4}{35}}[2, 6, 7, 11] + \sqrt{\frac{9}{35}}[2, 5, 8, 11] +$ $-\sqrt{\frac{3}{70}}[1, 4, 10, 11] - \sqrt{\frac{1}{35}}[1, 6, 8, 11] - \sqrt{\frac{32}{105}}[3, 4, 8, 11] + \sqrt{\frac{6}{35}}[4, 5, 6, 11]$
${}^2\text{H}_1^\downarrow$	$-\sqrt{\frac{1}{42}}[2, 6, 9, 11] + \sqrt{\frac{2}{21}}[2, 5, 10, 11] + \sqrt{\frac{1}{7}}[2, 7, 8, 11] - \sqrt{\frac{1}{42}}[1, 6, 10, 11] +$ $-\sqrt{\frac{1}{7}}[3, 4, 10, 11] - \sqrt{\frac{2}{21}}[4, 6, 7, 11] + \sqrt{\frac{8}{21}}[4, 5, 8, 11] - \sqrt{\frac{2}{21}}[3, 6, 8, 11]$
${}^2\text{H}_2^\downarrow$	$\sqrt{\frac{1}{210}}[1, 8, 10, 11] - \sqrt{\frac{8}{105}}[2, 7, 10, 11] + \sqrt{\frac{4}{35}}[3, 6, 10, 11] - \sqrt{\frac{9}{35}}[4, 5, 10, 11] +$ $+\sqrt{\frac{3}{70}}[2, 8, 9, 11] + \sqrt{\frac{1}{35}}[4, 6, 9, 11] - \sqrt{\frac{32}{105}}[4, 7, 8, 11] + \sqrt{\frac{6}{35}}[5, 6, 8, 11]$
${}^2\text{H}_3^\downarrow$	$-\sqrt{\frac{1}{30}}[2, 9, 10, 11] + \sqrt{\frac{1}{30}}[3, 8, 10, 11] - \sqrt{\frac{3}{10}}[4, 7, 10, 11] + \sqrt{\frac{3}{10}}[5, 6, 10, 11] +$ $+\sqrt{\frac{2}{15}}[4, 8, 9, 11] - \sqrt{\frac{1}{5}}[6, 7, 8, 11]$
${}^2\text{H}_4^\downarrow$	$-\sqrt{\frac{1}{5}}[4, 9, 10, 11] - \sqrt{\frac{8}{15}}[6, 7, 10, 11] + \sqrt{\frac{2}{15}}[5, 8, 10, 11] + \sqrt{\frac{2}{15}}[6, 8, 9, 11]$
${}^2\text{H}_5^\downarrow$	$-\sqrt{\frac{3}{5}}[6, 9, 10, 11] + \sqrt{\frac{2}{5}}[7, 8, 10, 11]$
	$[8, 9, 10, 11]$

Table A.3: Symmetry-adapted linear combinations of three-hole products, produced by our orthogonalization procedure.

symmetry-adapted state	linear combination of simple products
${}^4P_{-1}^{\uparrow\uparrow\uparrow}$	$-\sqrt{\frac{3}{5}}[1, 3, 9, 11] + \sqrt{\frac{2}{5}}[1, 5, 7, 11]$
${}^4P_0^{\uparrow\uparrow\uparrow}$	$-\sqrt{\frac{1}{5}}[1, 5, 9, 11] + \sqrt{\frac{4}{5}}[3, 5, 7, 11]$
${}^4P_1^{\uparrow\uparrow\uparrow}$	$-\sqrt{\frac{3}{5}}[1, 7, 9, 11] + \sqrt{\frac{2}{5}}[3, 5, 9, 11]$
${}^4P_{-1}^{\uparrow\uparrow\downarrow}$	$0.365148 [1,6,7,11] -0.447213 [1,4,9,11] -0.447213 [2,3,9,11] +0.365148 [2,5,7,11] +$ $+0.365149 [1,5,8,11] -0.447214 [1,3,10,11]$
${}^4P_0^{\uparrow\uparrow\downarrow}$	$0.516397 [3,5,8,11] -0.258199 [1,6,9,11] +0.516398 [3,6,7,11] +0.516397 [4,5,7,11] +$ $-0.258199 [1,5,10,11] -0.258199 [2,5,9,11]$
${}^4P_1^{\uparrow\uparrow\downarrow}$	$-0.447213 [1,7,10,11] +0.365148 [3,5,10,11] -0.447213 [1,8,9,11] +0.365148 [3,6,9,11] +$ $+0.365149 [4,5,9,11] -0.447214 [2,7,9,11]$
${}^4P_{-1}^{\uparrow\downarrow\downarrow}$	$0.365148 [2,5,8,11] -0.447213 [2,3,10,11] -0.447213 [1,4,10,11] +0.365148 [1,6,8,11] +$ $+0.365148 [2,6,7,11] -0.447213 [2,4,9,11]$
${}^4P_0^{\uparrow\downarrow\downarrow}$	$0.516397 [4,6,7,11] -0.258199 [2,5,10,11] +0.516397 [4,5,8,11] +0.516397 [3,6,8,11] +$ $-0.258199 [2,6,9,11] -0.258199 [1,6,10,11]$
${}^4P_1^{\uparrow\downarrow\downarrow}$	$-0.447213 [2,8,9,11] +0.365148 [4,6,9,11] -0.447213 [2,7,10,11] +0.365148 [4,5,10,11] +$ $+0.365148 [3,6,10,11] -0.447213 [1,8,10,11]$
${}^4P_{-1}^{\downarrow\downarrow\downarrow}$	$-\sqrt{\frac{3}{5}}[2, 4, 10, 11] + \sqrt{\frac{2}{5}}[2, 6, 8, 11]$
${}^4P_0^{\downarrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{5}}[2, 6, 10, 11] + \sqrt{\frac{4}{5}}[4, 6, 8, 11]$
${}^4P_1^{\downarrow\downarrow\downarrow}$	$-\sqrt{\frac{3}{5}}[2, 8, 10, 11] + \sqrt{\frac{2}{5}}[4, 6, 10, 11]$
${}^4F_{-3}^{\uparrow\uparrow\uparrow}$	$[1,3,5,11]$
${}^4F_{-2}^{\uparrow\uparrow\uparrow}$	$[1,3,7,11]$
${}^4F_{-1}^{\uparrow\uparrow\uparrow}$	$\sqrt{\frac{2}{5}}[1, 3, 9, 11] + \sqrt{\frac{3}{5}}[1, 5, 7, 11]$
${}^4F_0^{\uparrow\uparrow\uparrow}$	$-\sqrt{\frac{4}{5}}[1, 5, 9, 11] - \sqrt{\frac{1}{5}}[3, 5, 7, 11]$
${}^4F_1^{\uparrow\uparrow\uparrow}$	$-\sqrt{\frac{2}{5}}[1, 7, 9, 11] - \sqrt{\frac{3}{5}}[3, 5, 9, 11]$
${}^4F_2^{\uparrow\uparrow\uparrow}$	$-[3,7,9,11]$
${}^4F_3^{\uparrow\uparrow\uparrow}$	$[5,7,9,11]$
${}^4F_{-3}^{\uparrow\uparrow\downarrow}$	$\sqrt{\frac{1}{3}}[1, 4, 5, 11] + \sqrt{\frac{1}{3}}[1, 3, 6, 11] + \sqrt{\frac{1}{3}}[2, 3, 5, 11]$
${}^4F_{-2}^{\uparrow\uparrow\downarrow}$	$\sqrt{\frac{1}{3}}[1, 4, 7, 11] + \sqrt{\frac{1}{3}}[1, 3, 8, 11] + \sqrt{\frac{1}{3}}[2, 3, 7, 11]$
${}^4F_{-1}^{\uparrow\uparrow\downarrow}$	$0.365148 [1,4,9,11] +0.447213 [1,6,7,11] +0.447214 [1,5,8,11] +0.365148 [1,3,10,11] +$ $+0.365148 [2,3,9,11] +0.447214 [2,5,7,11]$
${}^4F_0^{\uparrow\uparrow\downarrow}$	$0.516398 [1,6,9,11] +0.516398 [2,5,9,11] +0.258199 [4,5,7,11] +0.516398 [1,5,10,11] +$ $+0.258199 [3,5,8,11] +0.258199 [3,6,7,11]$
${}^4F_1^{\uparrow\uparrow\downarrow}$	$-0.365148 [1,8,9,11] -0.447213 [3,6,9,11] -0.447214 [4,5,9,11] -0.365148 [2,7,9,11]$ $-0.365148 [1,7,10,11] -0.447214 [3,5,10,11]$
${}^4F_2^{\uparrow\uparrow\downarrow}$	$-\sqrt{\frac{1}{3}}[3, 8, 9, 11] - \sqrt{\frac{1}{3}}[4, 7, 9, 11] - \sqrt{\frac{1}{3}}[3, 7, 10, 11]$
${}^4F_3^{\uparrow\uparrow\downarrow}$	$-\sqrt{\frac{1}{3}}[5, 8, 9, 11] - \sqrt{\frac{1}{3}}[6, 7, 9, 11] - \sqrt{\frac{1}{3}}[5, 7, 10, 11]$

Table A.3: Symmetry-adapted linear combinations of three-hole products, produced by our orthogonalization procedure.

symmetry-adapted state	linear combination of simple products
${}^4F_{-3}^{\uparrow\downarrow\downarrow}$	$\sqrt{\frac{1}{3}}[2, 3, 6, 11] + \sqrt{\frac{1}{3}}[2, 4, 5, 11] + \sqrt{\frac{1}{3}}[1, 4, 6, 11]$
${}^4F_{-2}^{\uparrow\downarrow\downarrow}$	$\sqrt{\frac{1}{3}}[2, 3, 8, 11] + \sqrt{\frac{1}{3}}[2, 4, 7, 11] + \sqrt{\frac{1}{3}}[1, 4, 8, 11]$
${}^4F_{-1}^{\uparrow\downarrow\downarrow}$	$0.365148 [2,3,10,11] + 0.447213 [2,5,8,11] + 0.447213 [2,6,7,11] + 0.365148 [2,4,9,11] +$ $+ 0.365148 [1,4,10,11] + 0.447213 [1,6,8,11]$
${}^4F_0^{\uparrow\downarrow\downarrow}$	$0.516398 [2,5,10,11] + 0.516398 [1,6,10,11] + 0.258199 [3,6,8,11] + 0.516398 [2,6,9,11] +$ $+ 0.258199 [4,6,7,11] + 0.258199 [4,5,8,11]$
${}^4F_1^{\uparrow\downarrow\downarrow}$	$-0.365148 [2,7,10,11] - 0.447213 [4,5,10,11] - 0.447213 [3,6,10,11] - 0.365148 [1,8,10,11] +$ $- 0.365148 [2,8,9,11] - 0.447213 [4,6,9,11]$
${}^4F_2^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{3}}[4, 7, 10, 11] - \sqrt{\frac{1}{3}}[3, 8, 10, 11] - \sqrt{\frac{1}{3}}[4, 8, 9, 11]$
${}^4F_3^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{3}}[6, 7, 10, 11] - \sqrt{\frac{1}{3}}[5, 8, 10, 11] - \sqrt{\frac{1}{3}}[6, 8, 9, 11]$
${}^4F_{-3}^{\downarrow\downarrow\downarrow}$	$[2,4,6,11]$
${}^4F_{-2}^{\downarrow\downarrow\downarrow}$	$[2,4,8,11]$
${}^4F_{-1}^{\downarrow\downarrow\downarrow}$	$\sqrt{\frac{2}{5}}[2, 4, 10, 11] + \sqrt{\frac{3}{5}}[2, 6, 8, 11]$
${}^4F_0^{\downarrow\downarrow\downarrow}$	$-\sqrt{\frac{4}{5}}[2, 6, 10, 11] - \sqrt{\frac{1}{5}}[4, 6, 8, 11]$
${}^4F_1^{\downarrow\downarrow\downarrow}$	$-\sqrt{\frac{2}{5}}[2, 8, 10, 11] - \sqrt{\frac{3}{5}}[4, 6, 10, 11]$
${}^4F_2^{\downarrow\downarrow\downarrow}$	$-[4,8,10,11]$
${}^4F_3^{\downarrow\downarrow\downarrow}$	$[6,8,10,11]$

The other half of the symmetry-adapted three-hole states is obtained from the states depicted in Tables A.2 and A.3 by substituting the 4s hole [11] by the one of the opposite spin, i.e. [12].

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^1S(1)$	$-0.316228 [1,3,8,10] + 0.316228 [1,5,6,10] + 0.316228 [1,4,7,10] - 0.316228 [1,2,9,10] +$ $-0.316228 [3,5,6,8] - 0.316228 [3,4,7,8] + 0.316228 [2,3,8,9] + 0.316228 [4,5,6,7] +$ $-0.316228 [2,5,6,9] - 0.316228 [2,4,7,9]$
${}^1S(2)$	$-0.207019 [1,5,6,10] - 0.338062 [1,6,7,8] - 0.338062 [3,4,5,10] + 0.069006 [3,4,7,8] +$ $+ 0.338062 [3,4,6,9] + 0.338062 [2,5,7,8] + 0.207019 [2,5,6,9] + 0.207019 [1,3,8,10] +$ $- 0.276026 [1,4,8,9] - 0.207020 [3,5,6,8] + 0.207020 [4,5,6,7] - 0.276026 [2,3,7,10] +$ $+ 0.207019 [2,4,7,9] + 0.069007 [1,4,7,10] - 0.345033 [1,2,9,10] + 0.069007 [2,3,8,9]$
${}^1D_{-2}(1)$	$-0.377964 [1,3,4,10] - 0.308607 [1,2,5,10] + 0.308607 [1,3,6,8] - 0.308607 [2,3,5,8] +$ $- 0.377964 [3,4,5,6] - 0.308607 [1,4,6,7] + 0.308607 [2,4,5,7] + 0.308607 [1,2,6,9] +$ $+ 0.377964 [2,3,4,9]$
${}^1D_{-1}(1)$	$-0.154303 [1,3,6,10] + 0.154303 [1,4,5,10] - 0.377964 [1,2,7,10] - 0.154303 [3,4,5,8] +$ $- 0.377964 [2,3,7,8] + 0.377964 [1,5,6,8] - 0.377964 [2,5,6,7] + 0.377964 [1,4,7,8] +$ $+ 0.154303 [3,4,6,7] + 0.377964 [1,2,8,9] + 0.154303 [2,3,6,9] - 0.154303 [2,4,5,9]$
${}^1D_0(1)$	$-0.154303 [1,3,8,10] + 0.154303 [1,4,7,10] - 0.617213 [1,2,9,10] + 0.462910 [3,5,6,8] +$ $+ 0.308607 [3,4,7,8] + 0.154303 [2,3,8,9] - 0.462910 [4,5,6,7] - 0.154303 [2,4,7,9]$
${}^1D_1(1)$	$-0.154303 [1,5,8,10] + 0.154303 [1,6,7,10] - 0.377964 [1,4,9,10] - 0.154303 [3,6,7,8] +$ $- 0.377964 [3,4,8,9] + 0.377964 [3,5,6,10] - 0.377964 [4,5,6,9] + 0.377964 [3,4,7,10] +$ $+ 0.154303 [4,5,7,8] + 0.377964 [2,3,9,10] + 0.154303 [2,5,8,9] - 0.154303 [2,6,7,9]$
${}^1D_2(1)$	$-0.377964 [1,7,8,10] - 0.308607 [1,6,9,10] + 0.308607 [3,5,8,10] - 0.308607 [3,6,8,9] +$ $- 0.377964 [5,6,7,8] - 0.308607 [4,5,7,10] + 0.308607 [4,6,7,9] + 0.308607 [2,5,9,10] +$ $+ 0.377964 [2,7,8,9]$
${}^1D_{-2}(2)$	$0.218218 [1,4,6,7] + 0.218218 [1,2,6,9] - 0.267261 [1,3,4,10] + 0.534522 [3,4,5,6] +$ $+ 0.267261 [2,3,4,9] - 0.218218 [1,2,5,10] + 0.218218 [2,3,5,8] - 0.218218 [1,4,5,8] +$ $+ 0.534523 [1,2,7,8] - 0.218218 [2,3,6,7]$
${}^1D_{-1}(2)$	$-0.327327 [1,3,6,10] + 0.267262 [1,5,6,8] + 0.436436 [1,4,6,9] + 0.218218 [3,4,5,8] +$ $- 0.218218 [3,4,6,7] + 0.436436 [2,3,5,10] - 0.267262 [2,5,6,7] - 0.327327 [2,4,5,9] +$ $- 0.267261 [1,2,8,9] - 0.109109 [2,3,6,9] - 0.109109 [1,4,5,10] + 0.267261 [1,2,7,10]$
${}^1D_0(2)$	$-0.267261 [1,6,7,8] - 0.267261 [3,4,5,10] + 0.436436 [3,4,7,8] + 0.267261 [3,4,6,9] +$ $+ 0.267261 [2,5,7,8] - 0.327327 [1,3,8,10] + 0.218218 [1,4,8,9] + 0.218218 [2,3,7,10] +$ $- 0.327327 [2,4,7,9] + 0.109109 [1,4,7,10] + 0.436436 [1,2,9,10] + 0.109109 [2,3,8,9]$
${}^1D_1(2)$	$-0.327327 [1,5,8,10] + 0.436436 [1,6,8,9] + 0.267262 [3,5,6,10] + 0.218218 [3,6,7,8] +$ $- 0.218218 [4,5,7,8] - 0.267262 [4,5,6,9] + 0.436436 [2,5,7,10] - 0.327327 [2,6,7,9] +$ $- 0.109109 [1,6,7,10] + 0.267261 [1,4,9,10] - 0.267261 [2,3,9,10] - 0.109109 [2,5,8,9]$
${}^1D_2(2)$	$-0.267261 [1,7,8,10] - 0.218218 [1,6,9,10] + 0.218218 [3,6,8,9] + 0.534522 [5,6,7,8] +$ $+ 0.218218 [4,5,7,10] + 0.218218 [2,5,9,10] + 0.267261 [2,7,8,9] - 0.218218 [3,6,7,10] +$ $+ 0.534523 [3,4,9,10] - 0.218218 [4,5,8,9]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^1F_{-3}$	$0.273861 [1,4,5,6] + 0.223607 [1,2,6,7] + 0.273861 [1,3,4,8] - 0.273861 [2,3,4,7] +$ $-0.223607 [1,2,5,8] - 0.273861 [2,3,5,6] + 0.547723 [1,2,3,10] - 0.547723 [1,2,4,9]$
${}^1F_{-2}$	$0.273861 [1,3,6,8] - 0.091287 [1,4,6,7] - 0.365148 [1,2,6,9] - 0.223607 [1,3,4,10] +$ $+0.447214 [3,4,5,6] + 0.223607 [2,3,4,9] + 0.365148 [1,2,5,10] - 0.091287 [2,3,5,8] +$ $+0.273861 [2,4,5,7] - 0.182574 [1,4,5,8] - 0.447214 [1,2,7,8] - 0.182574 [2,3,6,7]$
${}^1F_{-1}$	$0.353553 [1,5,6,8] - 0.288675 [1,4,6,9] + 0.288675 [3,4,5,8] - 0.288675 [3,4,6,7] +$ $-0.288675 [2,3,5,10] - 0.353553 [2,5,6,7] - 0.353553 [1,4,7,8] + 0.288675 [2,3,6,9] +$ $+0.288675 [1,4,5,10] + 0.353553 [2,3,7,8]$
1F_0	$-\frac{1}{2}[1,6,7,8] + \frac{1}{2}[3,4,5,10] - \frac{1}{2}[3,4,6,9] + \frac{1}{2}[2,5,7,8]$
1F_1	$-0.288675 [1,6,7,10] + 0.288675 [1,6,8,9] + 0.353553 [3,4,7,10] - 0.353553 [3,4,8,9] +$ $+0.288675 [2,5,7,10] - 0.288675 [2,5,8,9] - 0.353553 [3,5,6,10] - 0.288675 [3,6,7,8] +$ $+0.288675 [4,5,7,8] + 0.353553 [4,5,6,9]$
1F_2	$-0.365148 [1,6,9,10] + 0.447214 [3,4,9,10] + 0.365148 [2,5,9,10] + 0.223607 [1,7,8,10] +$ $+0.182574 [3,6,7,10] + 0.091287 [3,6,8,9] + 0.091287 [4,5,7,10] + 0.182574 [4,5,8,9] +$ $-0.223607 [2,7,8,9] - 0.273861 [3,5,8,10] - 0.447214 [5,6,7,8] - 0.273861 [4,6,7,9]$
1F_3	$-0.547723 [1,8,9,10] + 0.223607 [3,6,9,10] - 0.223607 [4,5,9,10] + 0.547723 [2,7,9,10] +$ $-0.273861 [3,7,8,10] - 0.273861 [5,6,7,10] + 0.273861 [5,6,8,9] + 0.273861 [4,7,8,9]$
${}^1G_{-4}(1)$	$-\sqrt{\frac{1}{3}}[1,2,3,8] + \sqrt{\frac{1}{3}}[1,2,5,6] + \sqrt{\frac{1}{3}}[1,2,4,7]$
${}^1G_{-3}(1)$	$-\sqrt{\frac{1}{6}}[1,2,3,10] + \sqrt{\frac{1}{6}}[1,3,4,8] + \sqrt{\frac{1}{6}}[1,4,5,6] - \sqrt{\frac{1}{6}}[2,3,5,6] +$ $-\sqrt{\frac{1}{6}}[2,3,4,7] + \sqrt{\frac{1}{6}}[1,2,4,9]$
${}^1G_{-2}(1)$	$0.436436 [1,3,4,10] - 0.267261 [1,2,5,10] + 0.267261 [1,3,6,8] - 0.267261 [2,3,5,8] +$ $+0.436436 [3,4,5,6] - 0.267261 [1,4,6,7] + 0.267261 [2,4,5,7] + 0.267261 [1,2,6,9] +$ $-0.436436 [2,3,4,9]$
${}^1G_{-1}(1)$	$0.377964 [1,3,6,10] - 0.377964 [1,4,5,10] - 0.154303 [1,2,7,10] + 0.377964 [3,4,5,8] +$ $-0.154303 [2,3,7,8] + 0.154303 [1,5,6,8] - 0.154303 [2,5,6,7] + 0.154303 [1,4,7,8] +$ $-0.377964 [3,4,6,7] + 0.154303 [1,2,8,9] - 0.377964 [2,3,6,9] + 0.377964 [2,4,5,9]$
${}^1G_0(1)$	$0.207020 [1,3,8,10] + 0.483046 [1,5,6,10] - 0.207020 [1,4,7,10] - 0.138013 [1,2,9,10] +$ $-0.138013 [3,5,6,8] + 0.552052 [3,4,7,8] - 0.207020 [2,3,8,9] + 0.138013 [4,5,6,7] +$ $-0.483046 [2,5,6,9] + 0.207020 [2,4,7,9]$
${}^1G_1(1)$	$0.377964 [1,5,8,10] - 0.377964 [1,6,7,10] - 0.154303 [1,4,9,10] + 0.377964 [3,6,7,8] +$ $-0.154303 [3,4,8,9] + 0.154303 [3,5,6,10] - 0.154303 [4,5,6,9] + 0.154303 [3,4,7,10] +$ $-0.377964 [4,5,7,8] + 0.154303 [2,3,9,10] - 0.377964 [2,5,8,9] + 0.377964 [2,6,7,9]$
${}^1G_2(1)$	$0.436436 [1,7,8,10] - 0.267261 [1,6,9,10] + 0.267261 [3,5,8,10] - 0.267261 [3,6,8,9] +$ $+0.436436 [5,6,7,8] - 0.267261 [4,5,7,10] + 0.267261 [4,6,7,9] + 0.267261 [2,5,9,10] +$ $-0.436436 [2,7,8,9]$
${}^1G_3(1)$	$-\sqrt{\frac{1}{6}}[1,8,9,10] + \sqrt{\frac{1}{6}}[3,7,8,10] + \sqrt{\frac{1}{6}}[5,6,7,10] - \sqrt{\frac{1}{6}}[5,6,8,9] +$ $-\sqrt{\frac{1}{6}}[4,7,8,9] + \sqrt{\frac{1}{6}}[2,7,9,10]$
${}^1G_4(1)$	$-\sqrt{\frac{1}{3}}[3,8,9,10] + \sqrt{\frac{1}{3}}[5,6,9,10] + \sqrt{\frac{1}{3}}[4,7,9,10]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^1G_{-4}(2)$	$-0.639602 [1,3,4,6] -0.348156 [1,2,5,6] +0.639602 [2,3,4,5] -0.174078 [1,2,3,8] +$ $+0.174078 [1,2,4,7]$
${}^1G_{-3}(2)$	$0.307729 [1,4,5,6] +0.452267 [1,2,6,7] -0.430820 [1,3,4,8] +0.430820 [2,3,4,7] +$ $-0.452267 [1,2,5,8] -0.307729 [2,3,5,6] -0.123091 [1,2,3,10] +0.123091 [1,2,4,9]$
${}^1G_{-2}(2)$	$-0.282038 [1,3,6,8] +0.040292 [1,4,6,7] +0.322329 [1,2,6,9] -0.164487 [1,3,4,10] +$ $+0.328976 [3,4,5,6] +0.164487 [2,3,4,9] -0.322329 [1,2,5,10] +0.040292 [2,3,5,8] +$ $-0.282038 [2,4,5,7] +0.241747 [1,4,5,8] -0.592157 [1,2,7,8] +0.241747 [2,3,6,7]$
${}^1G_{-1}(2)$	$-0.227922 [1,3,6,10] -0.302408 [1,5,6,8] +0.170940 [1,4,6,9] +0.284901 [3,4,5,8] +$ $-0.284901 [3,4,6,7] +0.170940 [2,3,5,10] +0.302408 [2,5,6,7] -0.227922 [2,4,5,9] +$ $-0.162835 [1,4,7,8] +0.465242 [1,2,8,9] +0.056980 [2,3,6,9] +0.056980 [1,4,5,10] +$ $-0.465242 [1,2,7,10] +0.162835 [2,3,7,8]$
${}^1G_0(2)$	$-0.291287 [1,5,6,10] +0.076448 [1,6,7,8] +0.076448 [3,4,5,10] -0.076448 [3,4,6,9] +$ $-0.076448 [2,5,7,8] +0.291287 [2,5,6,9] -0.124837 [1,3,8,10] +0.374513 [1,4,8,9] +$ $-0.291288 [3,5,6,8] +0.291288 [4,5,6,7] +0.374513 [2,3,7,10] -0.124837 [2,4,7,9] +$ $-0.249675 [1,4,7,10] -0.416125 [1,2,9,10] -0.249675 [2,3,8,9] +0.166450 [3,4,7,8]$
${}^1G_1(2)$	$-0.227922 [1,5,8,10] +0.170940 [1,6,8,9] -0.302408 [3,5,6,10] +0.284901 [3,6,7,8] +$ $-0.284901 [4,5,7,8] +0.302408 [4,5,6,9] +0.170940 [2,5,7,10] -0.227922 [2,6,7,9] +$ $+0.056980 [1,6,7,10] -0.465242 [1,4,9,10] +0.162835 [3,4,8,9] -0.162835 [3,4,7,10] +$ $+0.465242 [2,3,9,10] +0.056980 [2,5,8,9]$
${}^1G_2(2)$	$-0.164487 [1,7,8,10] -0.322329 [1,6,9,10] -0.282038 [3,5,8,10] +0.040292 [3,6,8,9] +$ $+0.328976 [5,6,7,8] +0.040292 [4,5,7,10] -0.282038 [4,6,7,9] +0.322329 [2,5,9,10] +$ $+0.164487 [2,7,8,9] +0.241747 [3,6,7,10] -0.592157 [3,4,9,10] +0.241747 [4,5,8,9]$
${}^1G_3(2)$	$-0.430820 [3,7,8,10] -0.452267 [3,6,9,10] -0.307729 [5,6,8,9] +0.307729 [5,6,7,10] +$ $+0.452267 [4,5,9,10] +0.430820 [4,7,8,9] -0.123091 [1,8,9,10] +0.123091 [2,7,9,10]$
${}^1G_4(2)$	$-0.639602 [5,7,8,10] -0.348156 [5,6,9,10] +0.639602 [6,7,8,9] -0.174078 [3,8,9,10] +$ $+0.174078 [4,7,9,10]$
${}^1I_{-6}$	$[1,2,3,4]$
${}^1I_{-5}$	$\sqrt{\frac{1}{2}}[1,2,3,6] - \sqrt{\frac{1}{2}}[1,2,4,5]$
${}^1I_{-4}$	$0.369274 [1,2,3,8] +0.738549 [1,2,5,6] -0.369274 [1,2,4,7] -0.301511 [1,3,4,6] +$ $+0.301511 [2,3,4,5]$
${}^1I_{-3}$	$0.134840 [1,2,3,10] +0.495434 [1,2,5,8] -0.495434 [1,2,6,7] -0.134840 [1,2,4,9] +$ $-0.269680 [1,3,4,8] +0.404520 [1,4,5,6] -0.404520 [2,3,5,6] +0.269680 [2,3,4,7]$
${}^1I_{-2}$	$0.220193 [1,2,5,10] +0.404520 [1,2,7,8] -0.220193 [1,2,6,9] -0.134840 [1,3,4,10] +$ $+0.440386 [1,4,5,8] -0.330289 [1,4,6,7] -0.330289 [2,3,5,8] +0.440386 [2,3,6,7] +$ $+0.134840 [2,3,4,9] -0.110096 [1,3,6,8] +0.269680 [3,4,5,6] -0.110096 [2,4,5,7]$
${}^1I_{-1}$	$0.213201 [1,2,7,10] -0.213201 [1,2,8,9] +0.261116 [1,4,5,10] +0.426401 [1,4,7,8] +$ $-0.174078 [1,4,6,9] -0.174078 [2,3,5,10] -0.426401 [2,3,7,8] +0.261116 [2,3,6,9] +$ $-0.087039 [1,3,6,10] -0.213201 [1,5,6,8] +0.348155 [3,4,5,8] -0.348155 [3,4,6,7] +$ $+0.213201 [2,5,6,7] -0.087039 [2,4,5,9]$
1I_0	$0.131590 [1,2,9,10] +0.296078 [1,4,7,10] -0.263181 [1,4,8,9] -0.263181 [2,3,7,10] +$ $+0.296078 [2,3,8,9] -0.197386 [1,5,6,10] +0.241747 [1,6,7,8] +0.241747 [3,4,5,10] +$ $+0.526361 [3,4,7,8] -0.241747 [3,4,6,9] -0.241747 [2,5,7,8] +0.197386 [2,5,6,9] +$ $-0.032898 [1,3,8,10] -0.197386 [3,5,6,8] +0.197386 [4,5,6,7] -0.032898 [2,4,7,9]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
1I_1	0.213201 [1,4,9,10] -0.213201 [2,3,9,10] +0.261116 [1,6,7,10] -0.174078 [1,6,8,9] + +0.426401 [3,4,7,10] -0.426401 [3,4,8,9] -0.174078 [2,5,7,10] +0.261116 [2,5,8,9] + -0.087039 [1,5,8,10] -0.213201 [3,5,6,10] +0.348155 [3,6,7,8] -0.348155 [4,5,7,8] + +0.213201 [4,5,6,9] -0.087039 [2,6,7,9]
1I_2	0.220193 [1,6,9,10] +0.404520 [3,4,9,10] -0.220193 [2,5,9,10] -0.134840 [1,7,8,10] + +0.440386 [3,6,7,10] -0.330289 [3,6,8,9] -0.330289 [4,5,7,10] +0.440386 [4,5,8,9] + +0.134840 [2,7,8,9] -0.110096 [3,5,8,10] +0.269680 [5,6,7,8] -0.110096 [4,6,7,9]
1I_3	0.134840 [1,8,9,10] +0.495434 [3,6,9,10] -0.495434 [4,5,9,10] -0.134840 [2,7,9,10] + -0.269680 [3,7,8,10] +0.404520 [5,6,7,10] -0.404520 [5,6,8,9] +0.269680 [4,7,8,9]
1I_4	0.369274 [3,8,9,10] +0.738549 [5,6,9,10] -0.369274 [4,7,9,10] -0.301511 [5,7,8,10] + +0.301511 [6,7,8,9]
1I_5	$\sqrt{\frac{1}{2}}[5, 8, 9, 10] - \sqrt{\frac{1}{2}}[6, 7, 9, 10]$
1I_6	[7,8,9,10]
${}^3P_{-1}^{\uparrow\uparrow}(1)$	-0.365148 [1,2,8,10] +0.447214 [1,4,6,10] -0.447214 [3,4,6,8] -0.365148 [2,5,6,8] + -0.365148 [2,4,7,8] -0.447214 [2,4,6,9]
${}^3P_0^{\uparrow\uparrow}(1)$	0.258199 [1,4,8,10] +0.516398 [2,3,8,10] -0.516398 [2,5,6,10] +0.258199 [4,5,6,8] + -0.516398 [2,4,7,10] -0.258199 [2,4,8,9]
${}^3P_1^{\uparrow\uparrow}(1)$	0.447214 [1,6,8,10] -0.365148 [3,4,8,10] -0.365148 [4,5,6,10] -0.447214 [4,6,7,8] + -0.365148 [2,4,9,10] -0.447214 [2,6,8,9]
${}^3P_{-1}^{\uparrow\downarrow}(1)$	0.316228 [1,3,6,10] +0.316228 [1,4,5,10] -0.258199 [1,2,7,10] -0.316228 [3,4,5,8] + -0.258199 [2,3,7,8] -0.258199 [1,5,6,8] -0.258199 [2,5,6,7] -0.258199 [1,4,7,8] + -0.316228 [3,4,6,7] -0.258199 [1,2,8,9] -0.316228 [2,3,6,9] -0.316228 [2,4,5,9]
${}^3P_0^{\uparrow\downarrow}(1)$	0.547723 [1,3,8,10] -0.365148 [1,5,6,10] -0.182574 [1,4,7,10] +0.182574 [3,5,6,8] + +0.182574 [2,3,8,9] +0.182574 [4,5,6,7] -0.365148 [2,5,6,9] -0.547723 [2,4,7,9]
${}^3P_1^{\uparrow\downarrow}(1)$	0.316228 [1,5,8,10] +0.316228 [1,6,7,10] -0.258199 [1,4,9,10] -0.316228 [3,6,7,8] + -0.258199 [3,4,8,9] -0.258199 [3,5,6,10] -0.258199 [4,5,6,9] -0.258199 [3,4,7,10] + -0.316228 [4,5,7,8] -0.258199 [2,3,9,10] -0.316228 [2,5,8,9] -0.316228 [2,6,7,9]
${}^3P_{-1}^{\downarrow\downarrow}(1)$	0.447214 [1,3,5,10] -0.365148 [1,3,7,8] -0.365148 [1,5,6,7] -0.447214 [3,4,5,7] + -0.365148 [1,2,7,9] -0.447214 [2,3,5,9]
${}^3P_0^{\downarrow\downarrow}(1)$	0.258199 [1,3,7,10] +0.516398 [1,3,8,9] -0.516398 [1,5,6,9] +0.258199 [3,5,6,7] + -0.516398 [1,4,7,9] -0.258199 [2,3,7,9]
${}^3P_1^{\downarrow\downarrow}(1)$	-0.365148 [1,3,9,10] +0.447214 [1,5,7,10] -0.447214 [3,5,7,8] -0.365148 [3,5,6,9] + -0.365148 [3,4,7,9] -0.447214 [2,5,7,9]
${}^3P_{-1}^{\uparrow\uparrow}(2)$	0.239046 [1,4,6,10] -0.239046 [3,4,6,8] -0.358569 [2,4,5,10] +0.097590 [2,5,6,8] + -0.487949 [1,2,8,10] -0.358569 [2,3,6,10] +0.390360 [2,4,7,8] +0.478092 [2,4,6,9]
${}^3P_0^{\uparrow\uparrow}(2)$	-0.276026 [1,4,8,10] +0.507093 [3,4,6,10] +0.552052 [4,5,6,8] -0.069006 [2,4,7,10] + +0.507093 [2,6,7,8] +0.069006 [2,3,8,10] +0.138014 [2,5,6,10] +0.276026 [2,4,8,9]
${}^3P_1^{\uparrow\uparrow}(2)$	-0.478092 [1,6,8,10] +0.390360 [3,4,8,10] +0.097590 [4,5,6,10] -0.239046 [4,6,7,8] + -0.487949 [2,4,9,10] -0.239046 [2,6,8,9] +0.358569 [2,5,8,10] +0.358569 [2,6,7,10]

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^3P_{-1}^{\uparrow\downarrow}(2)$	-0.084516 [1,3,6,10] +0.069007 [1,5,6,8] +0.507093 [1,4,6,9] -0.169030 [3,4,5,8] + -0.169030 [3,4,6,7] -0.507093 [2,3,5,10] +0.069007 [2,5,6,7] +0.084516 [2,4,5,9] + +0.276027 [1,4,7,8] -0.345032 [1,2,8,9] +0.084516 [2,3,6,9] -0.084516 [1,4,5,10] + -0.345032 [1,2,7,10] +0.276027 [2,3,7,8]
${}^3P_0^{\uparrow\downarrow}(2)$	-0.146386 [1,3,8,10] +0.358568 [1,6,7,8] +0.390361 [3,5,6,8] +0.358568 [3,4,6,9] + +0.358568 [3,4,5,10] +0.390361 [4,5,6,7] +0.358568 [2,5,7,8] +0.146386 [2,4,7,9] + +0.097589 [1,5,6,10] -0.243975 [1,4,7,10] +0.243975 [2,3,8,9] +0.097589 [2,5,6,9]
${}^3P_1^{\uparrow\downarrow}(2)$	-0.084516 [1,5,8,10] -0.084516 [1,6,7,10] -0.345032 [1,4,9,10] -0.169030 [3,6,7,8] + +0.276027 [3,4,8,9] +0.069007 [3,5,6,10] +0.069007 [4,5,6,9] +0.276027 [3,4,7,10] + -0.169030 [4,5,7,8] -0.345032 [2,3,9,10] +0.084516 [2,5,8,9] +0.084516 [2,6,7,9] + +0.507093 [2,5,7,10] -0.507093 [1,6,8,9]
${}^3P_{-1}^{\downarrow\downarrow}(2)$	0.358569 [1,3,6,9] +0.097590 [1,5,6,7] -0.239046 [3,4,5,7] -0.239046 [2,3,5,9] + +0.390360 [1,3,7,8] +0.358569 [1,4,5,9] -0.487949 [1,2,7,9] -0.478092 [1,3,5,10]
${}^3P_0^{\downarrow\downarrow}(2)$	0.069006 [1,3,8,9] +0.507093 [1,5,7,8] +0.552052 [3,5,6,7] +0.507093 [3,4,5,9] + +0.276026 [2,3,7,9] -0.276026 [1,3,7,10] +0.138014 [1,5,6,9] -0.069006 [1,4,7,9]
${}^3P_1^{\downarrow\downarrow}(2)$	-0.487949 [1,3,9,10] +0.239046 [1,5,7,10] -0.239046 [3,5,7,8] +0.097590 [3,5,6,9] + +0.390360 [3,4,7,9] +0.478092 [2,5,7,9] -0.358569 [1,5,8,9] -0.358569 [1,6,7,9]
${}^3D_{-2}^{\uparrow\uparrow}$	0.436436 [1,2,6,10] -0.545545 [1,4,6,8] +0.534522 [2,3,4,10] +0.327327 [2,4,5,8] + -0.109109 [2,3,6,8] +0.327327 [2,4,6,7]
${}^3D_{-1}^{\uparrow\uparrow}$	-0.109109 [1,4,6,10] -0.436436 [3,4,6,8] -0.327327 [2,4,5,10] -0.534522 [2,5,6,8] + +0.534522 [1,2,8,10] +0.109109 [2,3,6,10] +0.327327 [2,4,6,9]
${}^3D_0^{\uparrow\uparrow}$	-0.534522 [3,4,6,10] +0.327327 [1,4,8,10] -0.327327 [2,4,7,10] +0.534522 [2,6,7,8] + -0.327327 [2,3,8,10] +0.327327 [2,4,8,9]
${}^3D_1^{\uparrow\uparrow}$	0.327327 [1,6,8,10] -0.327327 [2,5,8,10] +0.534522 [4,5,6,10] +0.109109 [2,6,7,10] + +0.436436 [4,6,7,8] -0.534522 [2,4,9,10] -0.109109 [2,6,8,9]
${}^3D_2^{\uparrow\uparrow}$	0.327327 [3,6,8,10] +0.327327 [4,5,8,10] -0.534522 [2,7,8,10] -0.109109 [4,6,7,10] + -0.436436 [2,6,9,10] -0.545545 [4,6,8,9]
${}^3D_{-2}^{\uparrow\downarrow}$	-0.462910 [1,3,6,8] -0.154303 [1,4,6,7] +0.308607 [1,2,6,9] +0.377964 [1,3,4,10] + +0.377964 [2,3,4,9] +0.308607 [1,2,5,10] +0.154303 [2,3,5,8] +0.462910 [2,4,5,7] + -0.154303 [1,4,5,8] +0.154303 [2,3,6,7]
${}^3D_{-1}^{\uparrow\downarrow}$	-0.377964 [1,5,6,8] +0.154303 [1,4,6,9] -0.308607 [3,4,5,8] -0.308607 [3,4,6,7] + -0.154303 [2,3,5,10] -0.377964 [2,5,6,7] +0.377964 [1,2,8,9] +0.308607 [2,3,6,9] + -0.308607 [1,4,5,10] +0.377964 [1,2,7,10]
${}^3D_0^{\uparrow\downarrow}$	0.377964 [1,6,7,8] -0.377964 [3,4,5,10] -0.377964 [3,4,6,9] +0.377964 [2,5,7,8] + +0.462910 [1,4,8,9] -0.462910 [2,3,7,10]
${}^3D_1^{\uparrow\downarrow}$	0.308607 [1,6,7,10] +0.154303 [1,6,8,9] -0.154303 [2,5,7,10] -0.308607 [2,5,8,9] + +0.377964 [3,5,6,10] +0.308607 [3,6,7,8] +0.308607 [4,5,7,8] +0.377964 [4,5,6,9] + -0.377964 [1,4,9,10] -0.377964 [2,3,9,10]
${}^3D_2^{\uparrow\downarrow}$	-0.377964 [1,7,8,10] +0.154303 [3,6,7,10] -0.154303 [3,6,8,9] +0.154303 [4,5,7,10] + -0.154303 [4,5,8,9] -0.377964 [2,7,8,9] -0.308607 [1,6,9,10] +0.462910 [3,5,8,10] + -0.462910 [4,6,7,9] -0.308607 [2,5,9,10]
${}^3D_{-2}^{\downarrow\downarrow}$	-0.327327 [1,3,6,7] +0.534522 [1,3,4,9] +0.436436 [1,2,5,9] +0.545545 [2,3,5,7] + -0.327327 [1,3,5,8] +0.109109 [1,4,5,7]
${}^3D_{-1}^{\downarrow\downarrow}$	0.327327 [1,3,6,9] -0.534522 [1,5,6,7] -0.436436 [3,4,5,7] +0.109109 [2,3,5,9] + -0.109109 [1,4,5,9] +0.534522 [1,2,7,9] -0.327327 [1,3,5,10]

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^3D_0^{\downarrow\downarrow}$	$-0.534522 [3,4,5,9] + 0.327327 [1,3,8,9] + 0.534522 [1,5,7,8] - 0.327327 [2,3,7,9] +$ $-0.327327 [1,3,7,10] + 0.327327 [1,4,7,9]$
${}^3D_1^{\downarrow\downarrow}$	$0.327327 [1,6,7,9] - 0.327327 [2,5,7,9] - 0.109109 [1,5,8,9] + 0.534522 [3,5,6,9] +$ $-0.534522 [1,3,9,10] + 0.109109 [1,5,7,10] + 0.436436 [3,5,7,8]$
${}^3D_2^{\downarrow\downarrow}$	$-0.534522 [1,7,8,9] - 0.327327 [3,6,7,9] - 0.327327 [4,5,7,9] - 0.436436 [1,5,9,10] +$ $+0.109109 [3,5,8,9] + 0.545545 [3,5,7,10]$
${}^3F_{-3}^{\uparrow\uparrow}(1)$	$-\sqrt{\frac{1}{3}}[1, 2, 4, 10] - \sqrt{\frac{1}{3}}[2, 3, 4, 8] - \sqrt{\frac{1}{3}}[2, 4, 5, 6]$
${}^3F_{-2}^{\uparrow\uparrow}(1)$	$-\sqrt{\frac{1}{3}}[1, 2, 6, 10] - \sqrt{\frac{1}{3}}[2, 3, 6, 8] + \sqrt{\frac{1}{3}}[2, 4, 6, 7]$
${}^3F_{-1}^{\uparrow\uparrow}(1)$	$-0.447214 [1,2,8,10] - 0.365148 [1,4,6,10] + 0.365148 [3,4,6,8] - 0.447214 [2,5,6,8] +$ $-0.447214 [2,4,7,8] + 0.365148 [2,4,6,9]$
${}^3F_0^{\uparrow\uparrow}(1)$	$-0.516398 [1,4,8,10] + 0.258199 [2,3,8,10] - 0.258199 [2,5,6,10] - 0.516398 [4,5,6,8] +$ $-0.258199 [2,4,7,10] + 0.516398 [2,4,8,9]$
${}^3F_1^{\uparrow\uparrow}(1)$	$-0.365148 [1,6,8,10] - 0.447214 [3,4,8,10] - 0.447214 [4,5,6,10] + 0.365148 [4,6,7,8] +$ $-0.447214 [2,4,9,10] + 0.365148 [2,6,8,9]$
${}^3F_2^{\uparrow\uparrow}(1)$	$-\sqrt{\frac{1}{3}}[3, 6, 8, 10] + \sqrt{\frac{1}{3}}[4, 6, 7, 10] - \sqrt{\frac{1}{3}}[2, 6, 9, 10]$
${}^3F_3^{\uparrow\uparrow}(1)$	$-\sqrt{\frac{1}{3}}[5, 6, 8, 10] - \sqrt{\frac{1}{3}}[4, 7, 8, 10] - \sqrt{\frac{1}{3}}[2, 8, 9, 10]$
${}^3F_{-3}^{\uparrow\downarrow}(1)$	$-\sqrt{\frac{1}{6}}[1, 2, 3, 10] - \sqrt{\frac{1}{6}}[1, 3, 4, 8] - \sqrt{\frac{1}{6}}[1, 4, 5, 6] - \sqrt{\frac{1}{6}}[2, 3, 5, 6] +$ $-\sqrt{\frac{1}{6}}[2, 3, 4, 7] - \sqrt{\frac{1}{6}}[1, 2, 4, 9]$
${}^3F_{-2}^{\uparrow\downarrow}(1)$	$-\sqrt{\frac{1}{6}}[1, 2, 5, 10] - \sqrt{\frac{1}{6}}[1, 3, 6, 8] - \sqrt{\frac{1}{6}}[2, 3, 5, 8] + \sqrt{\frac{1}{6}}[1, 4, 6, 7] +$ $+\sqrt{\frac{1}{6}}[2, 4, 5, 7] - \sqrt{\frac{1}{6}}[1, 2, 6, 9]$
${}^3F_{-1}^{\uparrow\downarrow}(1)$	$-0.258199 [1,3,6,10] - 0.258199 [1,4,5,10] - 0.316228 [1,2,7,10] + 0.258199 [3,4,5,8] +$ $-0.316228 [2,3,7,8] - 0.316228 [1,5,6,8] - 0.316228 [2,5,6,7] - 0.316228 [1,4,7,8] +$ $+0.258199 [3,4,6,7] - 0.316228 [1,2,8,9] + 0.258199 [2,3,6,9] + 0.258199 [2,4,5,9]$
${}^3F_0^{\uparrow\downarrow}(1)$	$-0.182574 [1,3,8,10] - 0.182574 [1,5,6,10] - 0.547723 [1,4,7,10] - 0.365148 [3,5,6,8] +$ $+0.547723 [2,3,8,9] - 0.365148 [4,5,6,7] - 0.182574 [2,5,6,9] + 0.182574 [2,4,7,9]$
${}^3F_1^{\uparrow\downarrow}(1)$	$-0.258199 [1,5,8,10] - 0.258199 [1,6,7,10] - 0.316228 [1,4,9,10] + 0.258199 [3,6,7,8] +$ $-0.316228 [3,4,8,9] - 0.316228 [3,5,6,10] - 0.316228 [4,5,6,9] - 0.316228 [3,4,7,10] +$ $+0.258199 [4,5,7,8] - 0.316228 [2,3,9,10] + 0.258199 [2,5,8,9] + 0.258199 [2,6,7,9]$
${}^3F_2^{\uparrow\downarrow}(1)$	$-\sqrt{\frac{1}{6}}[1, 6, 9, 10] - \sqrt{\frac{1}{6}}[3, 5, 8, 10] - \sqrt{\frac{1}{6}}[3, 6, 8, 9] + \sqrt{\frac{1}{6}}[4, 5, 7, 10] +$ $+\sqrt{\frac{1}{6}}[4, 6, 7, 9] - \sqrt{\frac{1}{6}}[2, 5, 9, 10]$
${}^3F_3^{\uparrow\downarrow}(1)$	$-\sqrt{\frac{1}{6}}[1, 8, 9, 10] - \sqrt{\frac{1}{6}}[3, 7, 8, 10] - \sqrt{\frac{1}{6}}[5, 6, 7, 10] - \sqrt{\frac{1}{6}}[5, 6, 8, 9] +$ $-\sqrt{\frac{1}{6}}[4, 7, 8, 9] - \sqrt{\frac{1}{6}}[2, 7, 9, 10]$
${}^3F_{-3}^{\downarrow\downarrow}(1)$	$-\sqrt{\frac{1}{3}}[1, 3, 5, 6] - \sqrt{\frac{1}{3}}[1, 3, 4, 7] - \sqrt{\frac{1}{3}}[1, 2, 3, 9]$
${}^3F_{-2}^{\downarrow\downarrow}(1)$	$-\sqrt{\frac{1}{3}}[1, 3, 5, 8] + \sqrt{\frac{1}{3}}[1, 4, 5, 7] - \sqrt{\frac{1}{3}}[1, 2, 5, 9]$
${}^3F_{-1}^{\downarrow\downarrow}(1)$	$-0.365148 [1,3,5,10] - 0.447214 [1,3,7,8] - 0.447214 [1,5,6,7] + 0.365148 [3,4,5,7] +$ $-0.447214 [1,2,7,9] + 0.365148 [2,3,5,9]$
${}^3F_0^{\downarrow\downarrow}(1)$	$-0.516398 [1,3,7,10] + 0.258199 [1,3,8,9] - 0.258199 [1,5,6,9] - 0.516398 [3,5,6,7] +$ $-0.258199 [1,4,7,9] + 0.516398 [2,3,7,9]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^3F_1^{\downarrow\downarrow}(1)$	$-0.447214 [1,3,9,10] -0.365148 [1,5,7,10] +0.365148 [3,5,7,8] -0.447214 [3,5,6,9] +$ $-0.447214 [3,4,7,9] +0.365148 [2,5,7,9]$
${}^3F_2^{\downarrow\downarrow}(1)$	$-\sqrt{\frac{1}{3}}[1, 5, 9, 10] - \sqrt{\frac{1}{3}}[3, 5, 8, 9] + \sqrt{\frac{1}{3}}[4, 5, 7, 9]$
${}^3F_3^{\downarrow\downarrow}(1)$	$-\sqrt{\frac{1}{3}}[1, 7, 9, 10] - \sqrt{\frac{1}{3}}[3, 7, 8, 9] - \sqrt{\frac{1}{3}}[5, 6, 7, 9]$
${}^3F_{-3}^{\uparrow\uparrow}(2)$	$0.707107 [1,2,6,8] +0.288676 [2,3,4,8] +0.288676 [2,4,5,6] -0.577350 [1,2,4,10]$
${}^3F_{-2}^{\uparrow\uparrow}(2)$	$0.577350 [1,4,6,8] +0.707107 [2,3,4,10] -0.288674 [2,3,6,8] -0.288676 [2,4,6,7]$
${}^3F_{-1}^{\uparrow\uparrow}(2)$	$0.365149 [1,4,6,10] +0.547722 [3,4,6,8] -0.547722 [2,4,5,10] -0.223607 [2,5,6,8] +$ $+0.365149 [2,3,6,10] +0.223607 [2,4,7,8] -0.182574 [2,4,6,9]$
${}^3F_0^{\uparrow\uparrow}(2)$	$0.258198 [1,4,8,10] +0.316228 [3,4,6,10] +0.316228 [2,6,7,8] +0.516398 [2,5,6,10] +$ $-0.258198 [2,4,8,9] +0.258199 [2,3,8,10] -0.516398 [4,5,6,8] -0.258199 [2,4,7,10]$
${}^3F_1^{\uparrow\uparrow}(2)$	$0.182574 [1,6,8,10] +0.223607 [3,4,8,10] -0.223607 [4,5,6,10] +0.547722 [4,6,7,8] +$ $-0.365149 [2,6,8,9] +0.547722 [2,5,8,10] -0.365149 [2,6,7,10]$
${}^3F_2^{\uparrow\uparrow}(2)$	$0.288676 [3,6,8,10] +0.288674 [4,6,7,10] -0.577350 [4,6,8,9] +0.707107 [2,7,8,10]$
${}^3F_3^{\uparrow\uparrow}(2)$	$0.288676 [5,6,8,10] +0.288676 [4,7,8,10] +0.707107 [4,6,9,10] -0.577350 [2,8,9,10]$
${}^3F_{-3}^{\uparrow\downarrow}(2)$	$0.204125 [1,4,5,6] +0.500000 [1,2,6,7] +0.204125 [1,3,4,8] +0.204125 [2,3,4,7] +$ $+0.500000 [1,2,5,8] +0.204125 [2,3,5,6] -0.408248 [1,2,3,10] -0.408248 [1,2,4,9]$
${}^3F_{-2}^{\uparrow\downarrow}(2)$	$0.204125 [1,3,6,8] +0.204123 [1,4,6,7] +0.500000 [1,3,4,10] +0.500000 [2,3,4,9] +$ $-0.204123 [2,3,5,8] -0.204125 [2,4,5,7] +0.408248 [1,4,5,8] -0.408248 [2,3,6,7]$
${}^3F_{-1}^{\uparrow\downarrow}(2)$	$0.516398 [1,3,6,10] -0.158115 [1,5,6,8] +0.129099 [1,4,6,9] +0.387298 [3,4,5,8] +$ $+0.387298 [3,4,6,7] -0.129099 [2,3,5,10] -0.158115 [2,5,6,7] -0.516398 [2,4,5,9] +$ $+0.158113 [1,4,7,8] +0.129100 [2,3,6,9] -0.129100 [1,4,5,10] +0.158113 [2,3,7,8]$
${}^3F_0^{\uparrow\downarrow}(2)$	$0.365148 [1,3,8,10] +0.223607 [1,6,7,8] +0.223607 [3,4,6,9] +0.223607 [3,4,5,10] +$ $+0.223607 [2,5,7,8] -0.365148 [2,4,7,9] +0.365148 [1,5,6,10] +0.365148 [2,5,6,9] +$ $-0.365148 [3,5,6,8] -0.365148 [4,5,6,7]$
${}^3F_1^{\uparrow\downarrow}(2)$	$0.516398 [1,5,8,10] -0.129100 [1,6,7,10] +0.387298 [3,6,7,8] +0.158113 [3,4,8,9] +$ $-0.158115 [3,5,6,10] -0.158115 [4,5,6,9] +0.158113 [3,4,7,10] +0.387298 [4,5,7,8] +$ $+0.129100 [2,5,8,9] -0.516398 [2,6,7,9] +0.129099 [2,5,7,10] -0.129099 [1,6,8,9]$
${}^3F_2^{\uparrow\downarrow}(2)$	$0.204125 [3,5,8,10] +0.408248 [3,6,7,10] -0.408248 [4,5,8,9] -0.204125 [4,6,7,9] +$ $+0.204123 [4,5,7,10] +0.500000 [1,7,8,10] +0.500000 [2,7,8,9] -0.204123 [3,6,8,9]$
${}^3F_3^{\uparrow\downarrow}(2)$	$0.204125 [5,6,7,10] +0.500000 [4,5,9,10] +0.204125 [3,7,8,10] +0.204125 [4,7,8,9] +$ $+0.500000 [3,6,9,10] +0.204125 [5,6,8,9] -0.408248 [1,8,9,10] -0.408248 [2,7,9,10]$
${}^3F_{-3}^{\downarrow\downarrow}(2)$	$0.288676 [1,3,5,6] +0.288676 [1,3,4,7] +0.707107 [1,2,5,7] -0.577350 [1,2,3,9]$
${}^3F_{-2}^{\downarrow\downarrow}(2)$	$0.707107 [1,3,4,9] -0.577350 [2,3,5,7] +0.288676 [1,3,5,8] +0.288674 [1,4,5,7]$
${}^3F_{-1}^{\downarrow\downarrow}(2)$	$0.547722 [1,3,6,9] -0.223607 [1,5,6,7] +0.547722 [3,4,5,7] -0.365149 [2,3,5,9] +$ $+0.223607 [1,3,7,8] -0.365149 [1,4,5,9] +0.182574 [1,3,5,10]$
${}^3F_0^{\downarrow\downarrow}(2)$	$0.316228 [1,5,7,8] +0.316228 [3,4,5,9] -0.258198 [2,3,7,9] +0.258198 [1,3,7,10] +$ $+0.516398 [1,5,6,9] +0.258199 [1,3,8,9] -0.516398 [3,5,6,7] -0.258199 [1,4,7,9]$
${}^3F_1^{\downarrow\downarrow}(2)$	$0.365149 [1,5,7,10] +0.547722 [3,5,7,8] -0.223607 [3,5,6,9] +0.223607 [3,4,7,9] +$ $-0.182574 [2,5,7,9] +0.365149 [1,5,8,9] -0.547722 [1,6,7,9]$
${}^3F_2^{\downarrow\downarrow}(2)$	$0.577350 [3,5,7,10] -0.288674 [3,5,8,9] -0.288676 [4,5,7,9] +0.707107 [1,7,8,9]$
${}^3F_3^{\downarrow\downarrow}(2)$	$0.707107 [3,5,9,10] +0.288676 [3,7,8,9] +0.288676 [5,6,7,9] -0.577350 [1,7,9,10]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^3G_{-4}^{\uparrow\uparrow}$	-0.774597 [2,3,4,6] -0.632456 [1,2,4,8]
${}^3G_{-3}^{\uparrow\uparrow}$	-0.547723 [1,2,6,8] -0.223607 [2,3,4,8] +0.670820 [2,4,5,6] -0.447214 [1,2,4,10]
${}^3G_{-2}^{\uparrow\uparrow}$	-0.585540 [1,2,6,10] -0.292770 [1,4,6,8] +0.119523 [2,3,4,10] +0.585540 [2,4,5,8] + +0.146385 [2,3,6,8] -0.439155 [2,4,6,7]
${}^3G_{-1}^{\uparrow\uparrow}$	-0.414039 [1,4,6,10] -0.207020 [3,4,6,8] +0.207020 [2,4,5,10] -0.253546 [2,5,6,8] + -0.338062 [1,2,8,10] +0.414039 [2,3,6,10] +0.591608 [2,4,7,8] -0.207020 [2,4,6,9]
${}^3G_0^{\uparrow\uparrow}$	-0.462910 [3,4,6,10] -0.377964 [1,4,8,10] +0.377964 [2,4,7,10] +0.462910 [2,6,7,8] + +0.377964 [2,3,8,10] -0.377964 [2,4,8,9]
${}^3G_1^{\uparrow\uparrow}$	-0.207020 [1,6,8,10] -0.591608 [3,4,8,10] +0.207020 [2,5,8,10] +0.253546 [4,5,6,10] + +0.414039 [2,6,7,10] +0.207020 [4,6,7,8] +0.338062 [2,4,9,10] -0.414039 [2,6,8,9]
${}^3G_2^{\uparrow\uparrow}$	-0.439155 [3,6,8,10] +0.585540 [4,5,8,10] -0.119523 [2,7,8,10] +0.146385 [4,6,7,10] + +0.585540 [2,6,9,10] -0.292770 [4,6,8,9]
${}^3G_3^{\uparrow\uparrow}$	-0.670820 [5,6,8,10] +0.223607 [4,7,8,10] +0.447214 [2,8,9,10] +0.547723 [4,6,9,10]
${}^3G_4^{\uparrow\uparrow}$	0.774597 [6,7,8,10] +0.632456 [4,8,9,10]
${}^3G_{-4}^{\uparrow\downarrow}$	-0.547723 [1,3,4,6] -0.547723 [2,3,4,5] -0.447214 [1,2,3,8] -0.447214 [1,2,4,7]
${}^3G_{-3}^{\uparrow\downarrow}$	0.474342 [1,4,5,6] -0.387298 [1,2,6,7] -0.158114 [1,3,4,8] -0.158114 [2,3,4,7] + -0.387298 [1,2,5,8] +0.474342 [2,3,5,6] -0.316228 [1,2,3,10] -0.316228 [1,2,4,9]
${}^3G_{-2}^{\uparrow\downarrow}$	-0.103510 [1,3,6,8] -0.517549 [1,4,6,7] -0.414039 [1,2,6,9] +0.084515 [1,3,4,10] + +0.084515 [2,3,4,9] -0.414039 [1,2,5,10] +0.517549 [2,3,5,8] +0.103510 [2,4,5,7] + +0.207020 [1,4,5,8] -0.207020 [2,3,6,7]
${}^3G_{-1}^{\uparrow\downarrow}$	-0.179284 [1,5,6,8] -0.439155 [1,4,6,9] -0.146385 [3,4,5,8] -0.146385 [3,4,6,7] + +0.439155 [2,3,5,10] -0.179284 [2,5,6,7] +0.418330 [1,4,7,8] -0.239046 [1,2,8,9] + +0.146385 [2,3,6,9] -0.146385 [1,4,5,10] -0.239046 [1,2,7,10] +0.418330 [2,3,7,8]
${}^3G_0^{\uparrow\downarrow}$	0.327327 [1,6,7,8] -0.327327 [3,4,5,10] -0.327327 [3,4,6,9] +0.327327 [2,5,7,8] + -0.534522 [1,4,8,9] +0.534522 [2,3,7,10]
${}^3G_1^{\uparrow\downarrow}$	0.146385 [1,6,7,10] -0.439155 [1,6,8,9] -0.418330 [3,4,7,10] -0.418330 [3,4,8,9] + +0.439155 [2,5,7,10] -0.146385 [2,5,8,9] +0.179284 [3,5,6,10] +0.146385 [3,6,7,8] + +0.146385 [4,5,7,8] +0.179284 [4,5,6,9] +0.239046 [1,4,9,10] +0.239046 [2,3,9,10]
${}^3G_2^{\uparrow\downarrow}$	-0.084515 [1,7,8,10] -0.207020 [3,6,7,10] -0.517549 [3,6,8,9] +0.517549 [4,5,7,10] + +0.207020 [4,5,8,9] -0.084515 [2,7,8,9] +0.414039 [1,6,9,10] +0.103510 [3,5,8,10] + -0.103510 [4,6,7,9] +0.414039 [2,5,9,10]
${}^3G_3^{\uparrow\downarrow}$	0.316228 [1,8,9,10] +0.158114 [3,7,8,10] -0.474342 [5,6,7,10] -0.474342 [5,6,8,9] + +0.158114 [4,7,8,9] +0.316228 [2,7,9,10] +0.387298 [3,6,9,10] +0.387298 [4,5,9,10]
${}^3G_4^{\uparrow\downarrow}$	0.447214 [3,8,9,10] +0.547723 [5,7,8,10] +0.547723 [6,7,8,9] +0.447214 [4,7,9,10]
${}^3G_{-4}^{\downarrow\downarrow}$	-0.774597 [1,3,4,5] -0.632456 [1,2,3,7]
${}^3G_{-3}^{\downarrow\downarrow}$	0.670820 [1,3,5,6] -0.223607 [1,3,4,7] -0.547723 [1,2,5,7] -0.447214 [1,2,3,9]
${}^3G_{-2}^{\downarrow\downarrow}$	-0.585540 [1,3,6,7] +0.119523 [1,3,4,9] -0.585540 [1,2,5,9] +0.292770 [2,3,5,7] + +0.439155 [1,3,5,8] -0.146385 [1,4,5,7]
${}^3G_{-1}^{\downarrow\downarrow}$	-0.207020 [1,3,6,9] -0.253546 [1,5,6,7] -0.207020 [3,4,5,7] +0.414039 [2,3,5,9] + +0.591608 [1,3,7,8] -0.414039 [1,4,5,9] -0.338062 [1,2,7,9] +0.207020 [1,3,5,10]
${}^3G_0^{\downarrow\downarrow}$	-0.462910 [3,4,5,9] -0.377964 [1,3,8,9] +0.462910 [1,5,7,8] +0.377964 [2,3,7,9] + +0.377964 [1,3,7,10] -0.377964 [1,4,7,9]
${}^3G_1^{\downarrow\downarrow}$	-0.207020 [1,6,7,9] -0.591608 [3,4,7,9] +0.207020 [2,5,7,9] -0.414039 [1,5,8,9] + +0.253546 [3,5,6,9] +0.338062 [1,3,9,10] +0.414039 [1,5,7,10] +0.207020 [3,5,7,8]

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^3G_2^{\uparrow\downarrow}$	$-0.119523 [1,7,8,9] -0.585540 [3,6,7,9] +0.439155 [4,5,7,9] +0.585540 [1,5,9,10] +$ $-0.146385 [3,5,8,9] +0.292770 [3,5,7,10]$
${}^3G_3^{\uparrow\downarrow}$	$0.447214 [1,7,9,10] +0.223607 [3,7,8,9] -0.670820 [5,6,7,9] +0.547723 [3,5,9,10]$
${}^3G_4^{\uparrow\downarrow}$	$0.632456 [3,7,9,10] +0.774597 [5,7,8,9]$
${}^3H_{-5}^{\uparrow\uparrow}$	$[1,2,4,6]$
${}^3H_{-4}^{\uparrow\uparrow}$	$-0.774597 [1,2,4,8] +0.632456 [2,3,4,6]$
${}^3H_{-3}^{\uparrow\uparrow}$	$-0.365148 [1,2,4,10] -0.447214 [1,2,6,8] +0.730297 [2,3,4,8] -0.365148 [2,4,5,6]$
${}^3H_{-2}^{\uparrow\uparrow}$	$-0.365148 [1,2,6,10] -0.182574 [1,4,6,8] +0.447214 [2,3,4,10] +0.547723 [2,3,6,8] +$ $-0.547723 [2,4,5,8] +0.182574 [2,4,6,7]$
${}^3H_{-1}^{\uparrow\uparrow}$	$-0.169031 [1,2,8,10] -0.207020 [1,4,6,10] +0.552052 [2,3,6,10] -0.276026 [3,4,6,8] +$ $-0.414039 [2,4,5,10] +0.507093 [2,5,6,8] -0.338062 [2,4,7,8] +0.069007 [2,4,6,9]$
${}^3H_0^{\uparrow\uparrow}$	$-0.154303 [1,4,8,10] +0.308607 [2,3,8,10] -0.377964 [3,4,6,10] +0.617213 [2,5,6,10] +$ $+0.308607 [4,5,6,8] -0.308607 [2,4,7,10] -0.377964 [2,6,7,8] +0.154303 [2,4,8,9]$
${}^3H_1^{\uparrow\uparrow}$	$-0.069007 [1,6,8,10] -0.338062 [3,4,8,10] +0.414039 [2,5,8,10] +0.507093 [4,5,6,10] +$ $-0.552052 [2,6,7,10] -0.276026 [4,6,7,8] -0.169031 [2,4,9,10] +0.207020 [2,6,8,9]$
${}^3H_2^{\uparrow\uparrow}$	$-0.182574 [3,6,8,10] +0.547723 [4,5,8,10] +0.447214 [2,7,8,10] -0.547723 [4,6,7,10] +$ $-0.365148 [2,6,9,10] +0.182574 [4,6,8,9]$
${}^3H_3^{\uparrow\uparrow}$	$-0.365148 [5,6,8,10] +0.730297 [4,7,8,10] -0.365148 [2,8,9,10] -0.447214 [4,6,9,10]$
${}^3H_4^{\uparrow\uparrow}$	$0.632456 [6,7,8,10] -0.774597 [4,8,9,10]$
${}^3H_5^{\uparrow\uparrow}$	$-[6,8,9,10]$
${}^3H_{-5}^{\uparrow\downarrow}$	$-\sqrt{\frac{1}{2}}[1, 2, 3, 6] - \sqrt{\frac{1}{2}}[1, 2, 4, 5]$
${}^3H_{-4}^{\uparrow\downarrow}$	$-0.547723 [1,2,3,8] -0.547723 [1,2,4,7] +0.447214 [1,3,4,6] +0.447214 [2,3,4,5]$
${}^3H_{-3}^{\uparrow\downarrow}$	$-0.258199 [1,2,3,10] -0.316228 [1,2,5,8] -0.316228 [1,2,6,7] -0.258199 [1,2,4,9] +$ $+0.516398 [1,3,4,8] -0.258199 [1,4,5,6] -0.258199 [2,3,5,6] +0.516398 [2,3,4,7]$
${}^3H_{-2}^{\uparrow\downarrow}$	$-0.258199 [1,2,5,10] -0.258199 [1,2,6,9] +0.316228 [1,3,4,10] -0.516398 [1,4,5,8] +$ $+0.516398 [2,3,6,7] +0.316228 [2,3,4,9] +0.258199 [1,3,6,8] -0.258199 [2,4,5,7]$
${}^3H_{-1}^{\uparrow\downarrow}$	$-0.119523 [1,2,7,10] -0.119523 [1,2,8,9] -0.439155 [1,4,5,10] -0.239046 [1,4,7,8] +$ $-0.097590 [1,4,6,9] +0.097590 [2,3,5,10] -0.239046 [2,3,7,8] +0.439155 [2,3,6,9] +$ $+0.243975 [1,3,6,10] +0.358569 [1,5,6,8] -0.195180 [3,4,5,8] -0.195180 [3,4,6,7] +$ $+0.358569 [2,5,6,7] -0.243975 [2,4,5,9]$
${}^3H_0^{\uparrow\downarrow}$	$-0.327327 [1,4,7,10] +0.327327 [2,3,8,9] +0.436436 [1,5,6,10] -0.267261 [1,6,7,8] +$ $-0.267261 [3,4,5,10] -0.267261 [3,4,6,9] -0.267261 [2,5,7,8] +0.436436 [2,5,6,9] +$ $+0.109109 [1,3,8,10] +0.218218 [3,5,6,8] +0.218218 [4,5,6,7] -0.109109 [2,4,7,9]$
${}^3H_1^{\uparrow\downarrow}$	$-0.439155 [1,6,7,10] +0.097590 [1,6,8,9] -0.239046 [3,4,7,10] -0.239046 [3,4,8,9] +$ $-0.097590 [2,5,7,10] +0.439155 [2,5,8,9] +0.243975 [1,5,8,10] +0.358569 [3,5,6,10] +$ $-0.195180 [3,6,7,8] -0.195180 [4,5,7,8] +0.358569 [4,5,6,9] -0.243975 [2,6,7,9] +$ $-0.119523 [1,4,9,10] -0.119523 [2,3,9,10]$
${}^3H_2^{\uparrow\downarrow}$	$0.316228 [1,7,8,10] -0.516398 [3,6,7,10] +0.516398 [4,5,8,9] +0.316228 [2,7,8,9] +$ $-0.258199 [1,6,9,10] +0.258199 [3,5,8,10] -0.258199 [4,6,7,9] -0.258199 [2,5,9,10]$
${}^3H_3^{\uparrow\downarrow}$	$-0.258199 [1,8,9,10] +0.516398 [3,7,8,10] -0.258199 [5,6,7,10] -0.258199 [5,6,8,9] +$ $+0.516398 [4,7,8,9] -0.258199 [2,7,9,10] -0.316228 [3,6,9,10] -0.316228 [4,5,9,10]$
${}^3H_4^{\uparrow\downarrow}$	$-0.547723 [3,8,9,10] +0.447214 [5,7,8,10] +0.447214 [6,7,8,9] -0.547723 [4,7,9,10]$
${}^3H_5^{\uparrow\downarrow}$	$-\sqrt{\frac{1}{2}}[5, 8, 9, 10] - \sqrt{\frac{1}{2}}[6, 7, 9, 10]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^3H_{-5}^{\downarrow\downarrow}$	$-[1,2,3,5]$
${}^3H_{-4}^{\downarrow\downarrow}$	$-0.774597 [1,2,3,7] 0.632456 [1,3,4,5]$
${}^3H_{-3}^{\downarrow\downarrow}$	$-0.365148 [1,2,3,9] -0.447214 [1,2,5,7] +0.730297 [1,3,4,7] -0.365148 [1,3,5,6]$
${}^3H_{-2}^{\downarrow\downarrow}$	$-0.365148 [1,2,5,9] +0.447214 [1,3,4,9] -0.547723 [1,4,5,7] +0.182574 [2,3,5,7] +$ $+0.547723 [1,3,6,7] -0.182574 [1,3,5,8]$
${}^3H_{-1}^{\downarrow\downarrow}$	$-0.169031 [1,2,7,9] -0.552052 [1,4,5,9] +0.207020 [2,3,5,9] +0.414039 [1,3,6,9] +$ $+0.507093 [1,5,6,7] -0.276026 [3,4,5,7] -0.338062 [1,3,7,8] -0.069007 [1,3,5,10]$
${}^3H_0^{\downarrow\downarrow}$	$-0.308607 [1,4,7,9] +0.154303 [2,3,7,9] +0.617213 [1,5,6,9] -0.377964 [3,4,5,9] +$ $+0.308607 [1,3,8,9] -0.377964 [1,5,7,8] +0.308607 [3,5,6,7] -0.154303 [1,3,7,10]$
${}^3H_1^{\downarrow\downarrow}$	$-0.414039 [1,6,7,9] -0.338062 [3,4,7,9] +0.069007 [2,5,7,9] +0.552052 [1,5,8,9] +$ $+0.507093 [3,5,6,9] -0.169031 [1,3,9,10] -0.207020 [1,5,7,10] -0.276026 [3,5,7,8]$
${}^3H_2^{\downarrow\downarrow}$	$0.447214 [1,7,8,9] -0.547723 [3,6,7,9] +0.182574 [4,5,7,9] -0.365148 [1,5,9,10] +$ $+0.547723 [3,5,8,9] -0.182574 [3,5,7,10]$
${}^3H_3^{\downarrow\downarrow}$	$-0.365148 [1,7,9,10] +0.730297 [3,7,8,9] -0.365148 [5,6,7,9] -0.447214 [3,5,9,10]$
${}^3H_4^{\downarrow\downarrow}$	$-0.774597 [3,7,9,10] +0.632456 [5,7,8,9]$
${}^3H_5^{\downarrow\downarrow}$	$-[5,7,9,10]$
${}^5D_{-2}^{\uparrow\uparrow\uparrow}$	$-[1,3,5,7]$
${}^5D_{-1}^{\uparrow\uparrow\uparrow}$	$-[1,3,5,9]$
${}^5D_0^{\uparrow\uparrow\uparrow}$	$-[1,3,7,9]$
${}^5D_1^{\uparrow\uparrow\uparrow}$	$-[1,5,7,9]$
${}^5D_2^{\uparrow\uparrow\uparrow}$	$-[3,5,7,9]$
${}^5D_{-2}^{\uparrow\uparrow\downarrow}$	$-\frac{1}{2}[1,3,5,8] - \frac{1}{2}[1,3,6,7] - \frac{1}{2}[1,4,5,7] - \frac{1}{2}[2,3,5,7]$
${}^5D_{-1}^{\uparrow\uparrow\downarrow}$	$-\frac{1}{2}[1,3,6,9] - \frac{1}{2}[1,4,5,9] - \frac{1}{2}[1,3,5,10] - \frac{1}{2}[2,3,5,9]$
${}^5D_0^{\uparrow\uparrow\downarrow}$	$-\frac{1}{2}[1,3,8,9] - \frac{1}{2}[2,3,7,9] - \frac{1}{2}[1,3,7,10] - \frac{1}{2}[1,4,7,9]$
${}^5D_1^{\uparrow\uparrow\downarrow}$	$-\frac{1}{2}[1,5,7,10] - \frac{1}{2}[2,5,7,9] - \frac{1}{2}[1,5,8,9] - \frac{1}{2}[1,6,7,9]$
${}^5D_2^{\uparrow\uparrow\downarrow}$	$-\frac{1}{2}[3,5,7,10] - \frac{1}{2}[3,5,8,9] - \frac{1}{2}[3,6,7,9] - \frac{1}{2}[4,5,7,9]$
${}^5D_{-2}^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{6}}[2,3,5,8] - \sqrt{\frac{1}{6}}[1,4,6,7] - \sqrt{\frac{1}{6}}[1,3,6,8] - \sqrt{\frac{1}{6}}[1,4,5,8] +$ $-\sqrt{\frac{1}{6}}[2,3,6,7] - \sqrt{\frac{1}{6}}[2,4,5,7]$
${}^5D_{-1}^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{6}}[1,4,6,9] - \sqrt{\frac{1}{6}}[1,3,6,10] - \sqrt{\frac{1}{6}}[2,3,6,9] - \sqrt{\frac{1}{6}}[1,4,5,10] +$ $-\sqrt{\frac{1}{6}}[2,4,5,9] - \sqrt{\frac{1}{6}}[2,3,5,10]$
${}^5D_0^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{6}}[2,3,8,9] - \sqrt{\frac{1}{6}}[1,3,8,10] - \sqrt{\frac{1}{6}}[1,4,8,9] - \sqrt{\frac{1}{6}}[2,3,7,10] +$ $-\sqrt{\frac{1}{6}}[2,4,7,9] - \sqrt{\frac{1}{6}}[1,4,7,10]$
${}^5D_1^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{6}}[2,5,7,10] - \sqrt{\frac{1}{6}}[1,5,8,10] - \sqrt{\frac{1}{6}}[1,6,7,10] - \sqrt{\frac{1}{6}}[2,5,8,9] +$ $-\sqrt{\frac{1}{6}}[2,6,7,9] - \sqrt{\frac{1}{6}}[1,6,8,9]$
${}^5D_2^{\uparrow\downarrow\downarrow}$	$-\sqrt{\frac{1}{6}}[4,5,7,10] - \sqrt{\frac{1}{6}}[3,6,8,9] - \sqrt{\frac{1}{6}}[3,5,8,10] - \sqrt{\frac{1}{6}}[3,6,7,10] +$ $-\sqrt{\frac{1}{6}}[4,5,8,9] - \sqrt{\frac{1}{6}}[4,6,7,9]$
${}^5D_{-2}^{\uparrow\downarrow\downarrow}$	$-\frac{1}{2}[2,3,6,8] - \frac{1}{2}[2,4,5,8] - \frac{1}{2}[1,4,6,8] - \frac{1}{2}[2,4,6,7]$
${}^5D_{-1}^{\uparrow\downarrow\downarrow}$	$-\frac{1}{2}[1,4,6,10] - \frac{1}{2}[2,4,6,9] - \frac{1}{2}[2,3,6,10] - \frac{1}{2}[2,4,5,10]$
${}^5D_0^{\uparrow\downarrow\downarrow}$	$-\frac{1}{2}[2,3,8,10] - \frac{1}{2}[2,4,8,9] - \frac{1}{2}[1,4,8,10] - \frac{1}{2}[2,4,7,10]$

Table A.4: Symmetry-adapted linear combinations of four-hole products, produced by our numerical procedure.

symmetry-adapted state	linear combination of simple products
${}^5D_1^{\uparrow\downarrow\downarrow\downarrow}$	$-\frac{1}{2}[2, 5, 8, 10] - \frac{1}{2}[2, 6, 7, 10] - \frac{1}{2}[1, 6, 8, 10] - \frac{1}{2}[2, 6, 8, 9]$
${}^5D_2^{\uparrow\downarrow\downarrow\downarrow}$	$-\frac{1}{2}[4, 5, 8, 10] - \frac{1}{2}[4, 6, 7, 10] - \frac{1}{2}[3, 6, 8, 10] - \frac{1}{2}[4, 6, 8, 9]$
${}^5D_{-2}^{\downarrow\downarrow\downarrow\downarrow}$	$-[2, 4, 6, 8]$
${}^5D_{-1}^{\downarrow\downarrow\downarrow\downarrow}$	$-[2, 4, 6, 10]$
${}^5D_0^{\downarrow\downarrow\downarrow\downarrow}$	$-[2, 4, 8, 10]$
${}^5D_1^{\downarrow\downarrow\downarrow\downarrow}$	$-[2, 6, 8, 10]$
${}^5D_2^{\downarrow\downarrow\downarrow\downarrow}$	$-[4, 6, 8, 10]$

Table A.5: Corrections to the free-ion (spherical symmetry) energy levels resulting from the surface ligand field.

free-ion state	surface state	ligand-field correction	resulting energy [eV]
1S	A_1	$2\varepsilon_0$	9.4113
3P	A_1	$2\varepsilon_0 + \frac{14}{5}D_S$	2.8150
	E	$2\varepsilon_0 - \frac{7}{5}D_S$	3.0565
1D	A_1	$2\varepsilon_0 + \frac{6}{7}D_S + \frac{24}{7}D_q$	2.6999
	B_1	$2\varepsilon_0 - \frac{6}{7}D_S + \frac{24}{7}D_q + \frac{20}{7}D_U$	3.2499
	B_2	$2\varepsilon_0 - \frac{6}{7}D_S - \frac{16}{7}D_q - \frac{20}{7}D_U$	2.6784
	E_{α}, E_{β}	$2\varepsilon_0 - \frac{3}{7}D_S - \frac{16}{7}D_q$	3.1052
3F	B_1	$2\varepsilon_0 + 2D_q - 5D_U$	0.0000
	E_{α}, E_{β}	$2\varepsilon_0 - D_S + 2D_q + \frac{15}{4}D_U$	1.4400
	B_2	$2\varepsilon_0 + 12D_q + 5D_U$	1.0000
	A_2	$2\varepsilon_0 - \frac{4}{5}D_S - 6D_q$	1.3000
	E_{γ}, E_{δ}	$2\varepsilon_0 + \frac{2}{5}D_S - 6D_q - \frac{15}{4}D_U$	0.6385
1G	$A_{1\alpha}$	$2\varepsilon_0 + 4D_q + \frac{5}{3}D_U$	3.9453
	$A_{1\beta}$	$2\varepsilon_0 + \frac{8}{7}D_S + \frac{4}{7}D_q - \frac{5}{3}D_U$	3.5518
	B_1	$2\varepsilon_0 - \frac{8}{7}D_S + \frac{4}{7}D_q + \frac{15}{7}D_U$	4.2851
	A_2	$2\varepsilon_0 + 4D_S + 2D_q$	3.5680
	E_{α}, E_{β}	$2\varepsilon_0 - 2D_S + 2D_q + \frac{5}{4}D_U$	4.1105
	B_2	$2\varepsilon_0 - \frac{8}{7}D_S - \frac{26}{7}D_q - \frac{15}{7}D_U$	3.8566
	E_{γ}, E_{δ}	$2\varepsilon_0 + \frac{4}{7}D_S - \frac{26}{7}D_q - \frac{5}{4}D_U$	3.8991