Chapter 2
Stripe domains in thin films

2.1 Films with perpendicular anisotropy

In the first part of this chapter, we discuss the magnetization of films with perpendicular uniaxial anisotropy. The easy axis of the anisotropy is chosen parallel to the film normal and favors an out-of-plane magnetization. The tendency of the magnetization to rotate out-of-plane, however, is hindered by the presence of the magnetic charges on the surface of the film which contribute to the magnetostatic energy. As a consequence, the magnetostatic energy (or the shape anisotropy) is smallest when the magnetization lies in the plane of the film. The competition between uniaxial and shape anisotropy determines whether the magnetization tends to be perpendicular or parallel to the film normal, provided that the magnetization is uniform. The two quantities are compared by means of the quality factor

\[ Q = \frac{2k_u}{\mu_0 M_s^2} \]  

(2.1)

the ratio between the uniaxial and the shape anisotropy. The uniaxial anisotropy is a function of the thickness \( t \) of the film and it is given by the sum of the volume anisotropy constant \( k_b \) and of the surface anisotropy constant \( k_s \) divided by \( t \):

\[ k_u(t) = k_b + \frac{k_s}{t} \]  

(2.2)

If \( Q < 1 \), the shape anisotropy is dominant and the magnetization tends to lie in-plane. If \( Q > 1 \), the uniaxial anisotropy forces the anisotropy to lie out-of-plane.

So far we have seen that the tendency of the magnetization to lie in or out-of-plane is connected to the ratio of uniaxial and shape anisotropies. To understand whether the system shows magnetic domains or a single domain, we have to analyse the energy of these configurations. In general, magnetic domains occur when the gain in magnetostatic energy due to the domain structure is bigger than the energy required to form the domain walls.
2.2 Domains separated by walls of negligible width

In order to calculate the total energy of a stripe configuration, some studies [38, 40, 42] follow the pioneering work of Kittel [39] who considered a series of stripes of width $D$, with the magnetization pointing alternatively up and down (Fig. 2.1). The stripes are assumed uniformly magnetised and separated by a wall of width $\delta_w$ much smaller than the domains $D$.

![Figure 2.1: Series of stripe domains for a thick film, i.e., for thickness $t$ much bigger than the domain width $D$. The magnetization is uniform within each stripe and the wall width $\delta_w$ is negligible with respect to the domain width $D$.](image)

The relevant energy is given by the sum of the magnetostatic energy and of the wall energy. The total wall energy per unit area of the stripe domain configuration in Fig. 2.1 is given by

$$e_w = \gamma_w \frac{t}{D}$$  \hspace{1cm} (2.3)

where $\gamma_w = 4\sqrt{Ak}$ is the energy of a Bloch wall and $t/D$ is the total domain wall area [28]. In the limit of thick films, i.e., $t \gg D$, the magnetostatic field created by the charges on one surface of the slab does not interact with the field of the other surface. Therefore the value of magnetostatic energy is two times the value obtained for a single surface of the slab. The field created by the surface charges with alternating sign $\pm M_s$, is given by the Fourier expansion of a square-wave of amplitude $2\pi M_s^2$. The magnetostatic energy density per unit area for one single surface results [39]:

$$e_{ms} = 0.85M_s^2D$$  \hspace{1cm} (2.4)

The dependence of the domain width $D$ from the thickness of the slab $t$ is obtained by minimizing the energy, given by the sum of eq. (2.3) and (2.4), with respect to $D$. The result is that in the limit of thick films, the domain width grows with the root of the
2.2 Domains separated by walls of negligible width

If the thickness of the film is smaller or comparable to the domain width, i.e., \( t \leq D \), the two surfaces interact magnetostatically. The magnetic field \( \mathbf{H} \) is approximately the same of a film uniformly magnetized, except for regions of size \( t \) around the wall Fig. 2.2 [26].

Figure 2.2: Stripe domains in the limit of small thickness, i.e., \( t \ll D \). The stray field crosses the film, side to side, and it is similar to the stray field of a single domain state, uniformly magnetized out-of-plane. The difference is localized around the border of successive stripes; in this region the flux closure slightly reduces the magnetostatic energy with respect to the value of a single domain state. The arrows indicate the stray field direction.

In this case the magnetostatic energy is a function of the thickness \( t \) and it is found by expanding the magnetic potential in a double Fourier series given by [40, 42]:

\[
e_{ms} = \frac{16M_s^2 D}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{2n} \left( 1 - \exp(-\frac{2n\pi t}{D}) \right)
\]  

(2.5)

In the limit of thick films, the exponential of eq. (2.5) is negligible and the magnetostatic energy tends to the value given by Kittel, eq. (2.4). In the limit of thin films, i.e., \( t \ll D \), the exponential can be expanded to the first order, so that \( \exp(-\frac{2n\pi t}{D}) \approx 1 - 2n\pi t/D \) and eq. (2.5) becomes:

\[
e_{ms} = 2\pi M_s^2 t
\]  

(2.6)

This expression shows that in the limit of thin films, the magnetostatic energy reaches the value of a film of thickness \( t \), uniformly magnetized parallel to the normal. An analytical study of the system in the limit of thin films, has been performed by Kaplan and Gehring [38]. Using the Taylor expansion of \( t/D \) up to the second order, they give the expression for the magnetostatic energy:

\[
e_{ms} = 2\pi M_s^2 t(1 - 0.67 \frac{t}{D} + \frac{2t}{\pi D} \ln \frac{t}{D})
\]  

(2.7)

The comparison of this equation with eq. (2.6) shows that the difference in magnetostatic energy between the stripe-domains state and the single domain state is vanishing with the thickness \( t \). The minimization of the total energy, obtained after summation
of equations (2.3) and (2.5), gives the value of the domain width $D(t)$ as a function of the thickness of the film. The numerical solution is plotted in logarithmic scale in Fig. (2.3) [43].

$$D(t) = 0.95t \exp \left( \frac{\pi D_0}{2t} \right)$$

(2.8)

where $D_0 = \gamma_m / 2 \pi M_s^2$ is the characteristic dipolar length. Equation (2.8) can be inserted in the expression of the total energy that results [45]:

$$\epsilon_t = 2\pi M_s^2 t \left( 1 - 0.67 \exp \left( -\frac{\pi D_0}{2t} \right) \right)$$

(2.9)

The total energy of the single domain state is equal to its magnetostatic energy and it is given by eq. (2.6). In the limit of thick films, i.e., $t \gg D$, the gain in magnetostatic
energy due to the flux closure is sufficiently high to guarantee that the energy for a stripe
domain configuration is lower than for a single domain state. By decreasing the thickness
of the film, the gain in magnetostatic energy becomes comparable to the wall energy and
the domain formation is less efficient. In the limit of thin films, i.e., \( t \ll D \), the energy for
the stripe domain configuration is given by eq. (2.9). From this equation we notice that
for every thickness of the slab, the value of the total energy is smaller than for a single
domain state. However, the rapid increasing of the domain width, rules out the possibility
to have a multi-domain state in a real sample of finite dimensions below a certain critical
thickness \( t_c \), when the domain width becomes of the same order of the extension of the
thin film.

2.3 Domains separated by walls of finite width

So far we have considered a system with uniform magnetization within the magnetic
domain and with a wall negligible with respect to the domain width. In reality, the Bloch
wall between two domains has a finite width and this contributes to the total energy
of the system. For certain sets of the parameters of the system, the dimension of the
wall becomes comparable with respect to the domain size and the picture of domains
separated by narrow walls loses its meaning. In such a case, the magnetization profile
can be described by a sine-like function between two maximal values which are a function
of the domain width [46]. The goal of this paragraph is to describe films with uniform
perpendicular anisotropy taking into account the effect of Bloch walls of finite width
between domains of opposite out of the plane magnetization.

2.3.1 Different contributions of the magnetostatic energy

The magnetostatic energy of a sample of volume \( V \) is given by:

\[
F_{ms} = -2\pi \int_V \mathbf{M}(r) \cdot \mathbf{H}(r) \, dr
\]  

(2.10)

The dipolar field \( \mathbf{H}(r) \) is the result of the interactions of all the magnetic moments
distributed in the sample. Therefore the magnetostatic energy density is a non-local
quantity, depending on the distribution of all the magnetic charges upon the sample. On
the other hand, the anisotropy is a local quantity which depends on the symmetry of
the magnetic material considered. In thin films it is possible to write the magnetostatic
energy as a sum of an anisotropy-type energy density term, proportional to the area of
the film, plus a term describing the influence of the wall fine structure [47]. In this way it
is possible to evaluate the rotation of the magnetization which takes place in the region
of the wall.

With the exception of some particular cases, the integral (2.10) has no analytical
solution and the magnetostatic energy can be calculated only numerically. In the case of
Chapter 2. Stripe domains in thin films

a uniformly magnetised film, however, the magnetostatic energy density results:

\[ f_{ms} = 2\pi M_s^2 \cos^2 \theta \]  \hspace{1cm} (2.11)

where \( \theta \) is the angle between the magnetization and the normal of the film. If the film is magnetized parallel to the normal, \( \theta \) is zero and eq. (2.11) equals eq. (2.6). Since in a system with perpendicular uniaxial anisotropy the angular dependence of the anisotropy energy is of the same form as eq. (2.11), it is convenient to define the following effective anisotropy:

\[ k_{\text{eff}} = k_u - 2\pi M_s^2 \]  \hspace{1cm} (2.12)

so that the total energy density of the system results:

\[ f_t = c + k_{\text{eff}} \sin^2 \theta \]  \hspace{1cm} (2.13)

In this way the uniformly magnetized system is fully described by anisotropy-type local quantities. It is useful to underline that the magnetostatic energy, although absorbed in the effective anisotropy constant as shown in eq. (2.13), is still a non-local quantity, a function of the mutual interaction of all the spins distributed in the sample. In general the magnetization along the sample is not uniform and the magnetization can rotate forming a Bloch wall, as sketched in Fig. 2.4 for a thin film.

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**Figure 2.4:** Infinitely long slab of thickness \( t \) and width \( 2L \). The Bloch wall between the two domains has the width \( \delta_w = \pi \sqrt{\frac{A}{K}} \).

In case of a thin film not uniformly magnetized along the \( x \) direction the magnetostatic energy per unit length can be written in the following way [44]:

\[ f_{ms}(L, t) = 2\pi Lt \int M_z^2(x) dx + \frac{Lt^2}{4} \int \frac{dM_z(x)}{dx} \frac{dM_z(x')}{dx'} \ln \frac{|x - x'|}{t} dx dx' \]  \hspace{1cm} (2.14)

where \( L \) is the length of the unit cell. The first part of the equation corresponds to the anisotropy-like term. It is proportional to the area of the film and it contains the local magnetization \( M_z(x) \), which is integrated across the wall. The second term represents the correction to the magnetostatic energy due to the rotation of the magnetization, as
shown by the derivative under integration, and it is negative. In the case of uniform magnetization the derivative is zero and eq. (2.14) reduces to eq. (2.11). In the limit of vanishing thickness, i.e., $t \to 0$, only the first term remains and the value of the magnetostatic energy can be compared to the one of eq. (2.6). In both cases the demagnetizing factor is equal to one, but the value of the magnetostatic energy is different. In fact, while eq. (2.6) refers to a film uniformly magnetized out of plane, the first term of eq. (2.14) refers to a system with non uniform magnetization. In this latter case both the average value of the magnetic charges and the magnetostatic energy reduce. In general, as will be clear in section 2.3.3, the second term of eq. (2.14) must be considered to guarantee the existence of magnetic domains in systems with uniform uniaxial anisotropy. The goal of section 5.1 will be to show that systems with alternated anisotropy can have magnetic domains even neglecting the correction to the magnetostatic energy.

### 2.3.2 Thickness dependence of the domain wall width

Before we analyze the formation of a multi-domain state, we study the thickness dependence of a Bloch wall clarifying the role of the different contributions to the magnetostatic energy introduced in the last section.

The surface charges of opposite signs on the two sides of a Bloch wall generate a stray field whose intensity is a function of the thickness of the slab. In order to measure the effect of the stray field at first we calculate the energy and the wall profile in the case of vanishing thickness. In this case no stray field is created around the Bloch wall. Therefore the correction to the magnetostatic energy is zero and the system can be described by means of anisotropy-type quantities, as seen in section 2.3.1. This state can be obtained by assuming zero magnetostatic energy and the value given by eq. (2.12) for the anisotropy.

With reference to Fig. 2.4, we consider a Bloch wall of width $\delta_w$ dividing an infinite film [48] of thickness $t$ in two regions, oppositely magnetized. In the example, the exchange constant is $A=1.05 \times 10^6$ erg/cm$^3$, the uniaxial anisotropy constant $k_u=1.35 \times 10^7$ erg/cm$^3$, and the spontaneous magnetization is $M_s=1440$ emu/cm$^3$. The profile of the wall for different thickness is plotted in Fig. 2.5. The shape of the wall is determined by the minimization of the anisotropy energy, the exchange energy and the magnetostatic energy. In particular both the contributions of the magnetostatic energy introduced in section 2.3.1 play a role in the determination of the wall structure. In fact, on the one hand the anisotropy-type term tends to enlarge the wall in order to minimize the surface charge density (principle of charges avoiding), on the other hand the correction to the magnetostatic energy tends to narrow it because of the flux closure generated around the wall (see Fig. 2.2). The weight of the two contributions is a function of the thickness of the film as can be seen by defining the following general expression of the effective anisotropy:

$$k_{eff}(t) = k_u(t) - \alpha(t)2\pi M_s^2$$

(2.15)

The function $\alpha(t)$ measures the contribution of the correction to the magnetostatic energy.
and has the following limits:
\[
\lim_{t \to 0} \alpha(t) = 1 \quad \text{and} \quad \lim_{t \gg \delta_w} \alpha(t) = 0 \tag{2.16}
\]

The two values are obtained respectively for a film with vanishing thickness and for a thick film. When the thickness of the film tends to zero, the effect of the circular field created around the wall is small and the anisotropy-type term is sufficient to describe the magnetostatic energy. The wall profile obtained in such a case is drawn as a solid line in Fig. 2.5.

![Figure 2.5: Influence of the thickness \( t \) of the film on the width of the Bloch wall. To study the dependency of the magnetostatic energy, the uniaxial anisotropy constant \( k(t) \) is considered thickness independent. The dotted and the solid line are the lower and the upper limit for the wall width. The limits correspond to the bulk value (dotted line) and to the zero thickness film (solid line). By increasing the thickness of the film the influence of the circular field created by the surface charges around the Bloch wall increases blocking the expansion of the wall.](image)

In this case the magnetostatic energy tends to \( 2\pi M_s^2 \) and the width of the wall tends to
\[
\delta_w = \pi \sqrt{\frac{A}{k_{\text{eff}}}} \tag{2.17}
\]

This equation represents the upper limit of the wall width for a system with uniaxial anisotropy \( k_u \). The plot in grey in Fig. 2.5 represents the case of an ultrathin film with vanishing thickness, \( t = 0.3 \) nm, when the circular magnetic field plays still a negligible role. By increasing the thickness of the film the effect of the field becomes stronger and the wall reduces its extension, as we can see in the dashed profile of Fig. 2.5. In the
2.3 Domains separated by walls of finite width

limit of thick films, i.e., for \( t \gg d \), the width of the wall is given by equation 1.23, which represents the lower limit for a system with uniaxial anisotropy \( k_u \). Hence, by means of the function \( \alpha(t) \), the thickness dependence of the circular magnetic field created by the surface charges is described. The validity of this function goes beyond the limit of thin films for which eq. (2.14) has been calculated, but does not describe completely the thickness dependence of the effective anisotropy which are present as well in the uniaxial anisotropy term of eq. (2.15).

2.3.3 Transition single domain/multi-domain state

In this section, first we want to compare the energy of a single domain state with the one of a multi-domain state, in order to find the critical domain size of the transition for systems with finite walls width. The domain size at the transition is a function of the thickness as well as of the hardness of the material represented by \( Q \). Secondly we show that a multi domain state cannot exist if the dipolar interaction is neglected and only local quantities are considered to describe the system. Finally we compare the transition to the case of systems with negligible wall width.

In general, a multi-domain state is energetically favorable in comparison to a single domain state if the energy required to form a wall is lower than the gain in magnetostatic energy due to the stray field. The formation of a multi-domain state can be analyzed in simple terms by considering two domains, opposite magnetized and separated by a Bloch wall, as shown in Fig. 2.4, and comparing the energy obtained with the one of a single domain state. The energy of the wall is independent on the length \( L \) of the slab. As a consequence the energy to pay to form the multi-domain state is constant with the length \( L \). On the contrary, the gain in magnetostatic energy density per unit area increases with \( L \). Therefore, for a certain critical value of the length \( L_0 \), the energy of the multi-domain state becomes favorable.

The situation is well represented by the following equation, obtained by integration of eq. (2.14), using the profile of the Bloch wall \( M_s = M_t \tanh(x/\delta) \) [41, 44]:

\[
\frac{\Delta F}{Lt} = \gamma_w + \frac{\mu_0 M_s^2 t}{\pi} \ln \frac{2c_w \delta}{L} \tag{2.18}
\]

where \( c_w \approx 1.356 \) is a numerical factor.

Equation (2.18) gives the energy difference between single and multi-domain state; it is positive for \( L < L_0 \) and negative for \( L > L_0 \). The term responsible for the domain formation is the logarithm and comes from the non-local part of the magnetostatic energy written in eq. (2.14). This term, which has to be negative in order to reduce the value of the total energy, drives the transition by increasing \( L \), length of the slab. The critical length \( L_0 \), normalized to the exchange length \( l_{ex} \), is obtained from eq. (2.18) for \( \Delta F = 0 \):

\[
\frac{L_0}{l_{ex}} = \frac{1.9}{\sqrt{2Q - 1}} \exp \frac{8.9\sqrt{2Q - 1}}{l_{ex}} \tag{2.19}
\]
Chapter 2. Stripe domains in thin films

Figure 2.6: Thickness of the film versus $L_0$, the critical length for the transition to the multi-domain state. The plots are normalized to the exchange length $l_{ex}$ and refer to eq. (2.18) for different values of $Q$. With increasing $Q$ or decreasing $t$, $L_0$ shifts to higher values. In this way the drop in magnetostatic energy balances the increase of the wall energy.

The thickness of the film versus the critical length $L_0$, for different values of $Q$, is plotted in Fig. (2.6). As already shown in section 2.2 for the case of negligible wall width, the domain size increases rapidly with thickness decreasing, till the size of the sample is reached and the multi domain state cannot exist anymore. This happens because the gain in magnetostatic energy decreases with the thickness and therefore the length $L$ of the slab has to increase to allow domain formation.

The dependence of the domain size on the hardness of the material is clear if we consider the effective anisotropy, whose value increases with $Q$, as can be deduced from eq. (2.15). As a consequence the wall energy increases and, in order to obtain a multi domain configuration, the length $L$ of the slab has to become bigger so that the gain in magnetostatic energy can be comparable to the energy paid to form the wall.

Before comparing the different models introduced in the last sections, we show that the existence of the stray field created around the Bloch wall is essential to understand the formation of a multi-domain state. Let us consider once more a system with uniaxial perpendicular effective anisotropy, described only by means of the exchange energy and the anisotropy-type energy terms. If the system is uniformly magnetized out-of-plane, the exchange and the effective anisotropy energy are zero. As soon as a wall is formed and some of the magnetic moments are tilted away from the normal of the film, the effective anisotropy energy and the exchange energy are no more zero. Therefore, if the dipolar
interaction is not considered in its complete form, the energy of the system increases and
the single domain state is the lowest in energy. This shows that the complete non-local
dipolar interaction is the fundamental quantity to be considered to explain the formation
of a multi-domain state for a system with perpendicular uniaxial anisotropy. In Fig. 2.7 we
compare the energies of a multi-domain state with a single-domain state as a function of
the thickness, normalized to a fixed value of the domain width. The numerical calculation
is performed in the case of an anisotropy-type description (white dots in Fig. 2.7), when the
circular magnetic field is not considered, and in the case of total dipolar interaction (black
dots in Fig. 2.7). If the correction to the dipolar energy is not considered, the ratio of the
total energy is constant with the thickness and the single domain state has lower energy
than the multi-domain state. This happens because the magnetostatic energy density is
thickness independent for both states. If the correction is considered, the magnetostatic
energy density is no more thickness independent. As a consequence the total energy of
the multi-domain state decreases linearly with the thickness allowing the transition to the
multi-domain state.

The analysis carried out in this section has the purpose to determine the conditions
for which it is favorable to have a multi-domain configuration. To complete the picture,
we want to study the influence of the presence of the domain wall in the determination of
the domain size by comparing eq. (2.18) with eq. (2.8), calculated with the assumption of
uniform magnetization and negligible wall width. The two curves are plotted in Fig. 2.8,
Chapter 2. Stripe domains in thin films

Figure 2.8: $L_0$, critical length for the transition to the multi-domain state versus $t$, thickness of the film. The plots shown, normalized to the exchange length $l_{ex}$, are for a value of $Q=1.3$. The black plot refers to eq. (2.8), where the wall-width is neglected. The dashed plot refers to eq. (2.18), where the wall width is considered finite. The black points result from numeric simulation performed with periodic boundary conditions.

for $Q=1.3$. Fixing an arbitrary value of the thickness of the slab we notice that the domain size is significantly larger if the wall is included in the description. In this case the average perpendicular magnetization around the region of the wall is smaller than for a system with negligible wall width. As a consequence the gain in magnetostatic energy due to the flux closure is limited by the presence of the wall and the transition to the multi-domain state shifts to larger values of the domain size. Therefore, the assumptions of uniform magnetization and negligible wall width made in section 2.2, cause an overestimation of the gain in magnetostatic energy and a significant underestimation of the value of the domain size. To calculate correctly the size of the unit cell in the case of finite wall width, we have to bound the system periodically. In this way a new Bloch wall forms at the edge of the slab and the unit cell of the multi-domain state is given by the sum of the domain width and of the wall width. This choice, not considered in the derivation of eq. (2.18) [41], yields a further shift of the value of the domain size, as shown in Fig. 2.8. The difference is due to the presence of the Bloch wall at the edge of the slab and to the consequent reduction of the average magnetization around it.