Electronic Transport in Mesoscopic Systems

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Katja, I believe I have found the missing piece of my puzzle...
Abstract

The phase coherence of charge carriers gives rise to the unique transport properties of mesoscopic systems. This makes them interesting to study from a fundamental point of view, but also gives these small systems a possible future in nanoelectronics applications.

In the present work, a numerical method is implemented in order to contribute to the understanding of two-dimensional mesoscopic systems. The method allows for the calculation of a wide range of transport quantities, incorporating a complete description of both the charge and spin degrees of freedom of the electron. As such, it constitutes a valuable tool in the study of mesoscopic devices. This is illustrated by applying the numerics to three distinct problems.

First, the method gives an efficient means of simulating recent scanning probe experiments in which the coherent flow of electrons through a two-dimensional sample is visualized. This is done by measuring the conductance decrease of the sample as a function of the position of a perturbing probe. For electrons passing through a narrow constriction, the obtained flow visualizations show a separation of the current into several branches, which is in agreement with experimental observations. The influence of a magnetic field on these branches is studied, and the formation of cyclotron orbits at the sample edges is visualized, although only after a new measurement setup is proposed. Furthermore, a wealth of interference phenomena are present in the flow maps, illustrating the coherent nature of electrons in the system.

Second, the numerical scheme also permits a phenomenological modeling of phase breaking scattering centers in the sample. As an application of this model, the influence of phase randomizing processes on the transport characteristics of a four-contact ring is investigated.

Third, transport of electrons through a noncoplanar magnetic texture is studied, and a Hall effect is observed even in the absence of a net Lorentz force and without invoking any form of spin-orbit coupling. This Hall effect is due to the Berry phase picked up by electrons when their spin follows the local magnetization direction. Using numerics in simple magnetic texture models, both the limit where the spin follows the magnetization adiabatically and its nonadiabatic counterpart can be addressed, including the effect of disorder. By investigating the transition between both limits, an ongoing discussion in the literature about the relevant adiabaticity criterion in the diffusive regime is clarified.